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# Sampling procedures for inspection by attributes —

## Part 0: Introduction to the BS 6001 attribute sampling system

ICS 03.120.30

# Committees responsible for this British Standard

The preparation of this British Standard was entrusted to Technical Committee SS/5, Acceptance sampling schemes, upon which the following bodies were represented:

British Gas plc
British Standards Society
Clay Pipe Development Association Limited
Institute of Metal Finishing
Institute of Quality Assurance
Ministry of Defence
Royal Statistical Society

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# Contents

# National foreword

This Part of BS 6001 has been prepared by Technical Committee SS/5. It supersedes BS 6000:1972, which is withdrawn.

This standard is identical with ISO 2859-0 *Sampling procedures for inspection by attributes — Part 0: Introduction to the ISO 2859 attribute sampling system,* published by the International Organization for Standardization (ISO). As ISO 2859-0 was largely based on BS 6000:1972, this Part of BS 6001 is in effect a new edition of that British Standard. The change of designation was decided upon by the national Technical Committee responsible (SS/5, until recently designated QMS/15) in view of the fact that, like the other Parts of BS 6001, it deals exclusively with sampling for inspection by attributes.

NOTE Having regard to the fact that the Parts of BS 6002 are concerned with sampling for inspection by variables, it is considered that "BS 6000" is a more appropriate designation for a guide on acceptance sampling in general. This BS number has therefore now been allocated to the national adoption of ISO Technical Report ISO TR 8550 *Guide for the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots.*BS 6000 which is in preparation.

BS 6001 is published in the following Parts.

— *Part 0: Introduction to the BS 6001 attribute sampling system;*

— *Part 1: Specification for sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection;*

— *Part 2: Specification for sampling plans indexed by limiting quality (LQ) for isolated lot inspection;*

— *Part 3: Specification for skip-lot procedures;*

— *Part 4: Specification for sequential sampling plans.*

**Cross-references**

| International standard | Corresponding British Standard |
|---|---|
| | BS 6001 *Sampling procedures for inspection by attributes* |
| ISO 2859-1:1989 | Part 1:1991 *Specification for sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection* |
| | (Identical) |
| ISO 2859-2:1985 | Part 2:1993 *Specification for sampling plans indexed by limiting quality (LQ) for isolated lot inspection* |
| | (Identical) |
| ISO 2859-3:1991 | Part 3:1993 *Specification for skip-lot procedures* |
| | (Identical) |
| ISO 3951:1989 | BS 6002 *Sampling procedures for inspection by variables* |
| | Part 1:1993 *Specification for single sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection* |
| | (Identical) |
| ISO 8422:1991 | BS 6001 *Sampling procedures for inspection by attributes* |
| | Part 4:1994 *Specification for sequential sampling plans* |
| | (Identical) |
| ISO 8423:1991 | BS 6002 *Sampling procedures for inspection by variables* |

| | |
|---|---|
| | Part 4 *Specification for sequential sampling plans for percent nonconforming* |
| | Section 4.1:1994 *Known standard deviation* |
| | (Identical) |
| ISO TR 8550:1994 | [a]BS 6000:1996 *Guide for the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots* |
| | (Identical) |

[a] In preparation.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

**Compliance with a British Standard does not of itself confer immunity from legal obligations.**

**Summary of pages**

This document comprises a front cover, an inside front cover, pages i to vi, pages 1 to 58, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

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# Section 1. General

## 1.1 Scope

This part of ISO 2859 explains the terms used in acceptance sampling, describes the various schemes and plans, gives practical advice on sampling inspection and discusses some of the theoretical aspects.

Section 2 gives general information on methods of acceptance sampling inspection with particular reference to the sampling procedures and tables for inspection by attributes given in parts 1, 2 and 3 of ISO 2859 and in ISO 8422.

Section 3 extends the introduction to acceptance sampling given in Section 2 and amplifies the introductory text and instructions contained in ISO 2859-1, by giving detailed comments and examples to assist in using the method of sampling inspection that constitutes the ISO 2859-1 sampling system.

## 1.2 Normative references

The following standards contain provisions which, through reference in this text, constitute provisions of this part of ISO 2859. At the time of publication, the editions indicated were valid. All standards are subject to revision, and parties to agreements based on this part of ISO 2859 are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 2859-1:1989, *Sampling procedures for inspection by attributes — Part 1: Sampling plans indexed by acceptable quality level (AQL) for lot-by-lot inspection.*

ISO 2859-2:1985, *Sampling procedures for inspection by attributes — Part 2: Sampling plans indexed by limiting quality (LQ) for isolated lot inspection.*

ISO 2859-3:1991, *Sampling procedures for inspection by attributes — Part 3: Skip-lot sampling procedures.*

ISO 8422:1991, *Sequential sampling plans for inspection by attributes.*

ISO/TR 8550:1994, *Guide for the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots.*

## 1.3 Definitions

For the purposes of this part of ISO 2859, the definitions given in ISO 2859-1 and ISO 2859-3 apply.

# Section 2. General introduction to acceptance sampling

## 2.1 Aim of sampling inspection

A major aim of acceptance sampling inspection is to see that the producer submits lots at a quality which is at or better than a mutually agreed level, so that the consumer receives lots of a quality that is acceptable.

The producer may use these sampling procedures to assure that the quality level will be acceptable to the consumer. In all these procedures, it has to be recognized that the financial resources are not unlimited. The cost of the article has to reflect the cost of inspection as well as the cost of production.

A real effort should be made to ensure that a system is devised that clearly places responsibility for quality with the producer. Inspection can appear to divert the responsibility for quality from the producer to the inspector. This may happen whenever there is a feeling that the inspector is there to sort things out, so that, within limits, what happens in production will be caught by inspection. This feeling is completely misplaced and may result in hard work, high cost and poor quality for the consumer and the producer. The inspector has no means of inserting quality into a product if the producer has not done so.

## 2.2 Acceptance sampling

Acceptance sampling inspection has the merit of putting the responsibility for quality fairly and squarely where it belongs — with the producer. The inspector is no longer regarded as the person who sorts things out. The producer has to see that the quality of the product is right, otherwise there will be much trouble and expense with unacceptable lots. Sampling inspection can and should lead to less inspection work, lower cost and good quality for the consumer.

The sampling inspection schemes of parts 1, 2 and 3 of ISO 2859 and of ISO 8422 provide for quantification of the risks of accepting unsatisfactory product (known as "consumer's risk") and the risks of not accepting satisfactory product ("producer's risk") and for choosing a plan that allows no more risk than is acceptable.

In addition to the ISO sampling plans which are based on the mathematical theory of probability, there are several other practices:

a) sampling based on experience with the product, the process, the supplier and the consumer (see **2.2.1**);

b) *ad hoc* sampling, for example the inspection of a fixed percentage, or occasional random checks (see **2.2.2**);

c) 100 % inspection (see **2.2.3**);

d) other "sampling" practices (see **2.2.4**).

### 2.2.1 Statistical sampling

Sampling based on experience with the product, the process, the producer and the consumer can be statistically evaluated.

An example is the procedure described in ISO 2859-1 which uses a set of switching rules. When quality is very good, it is possible to go to reduced inspection. This provides a procedure where, if smaller samples are used, the producer's risk is reduced but the consumer's risk is increased. If experience is good, this is justifiable, particularly when the process average has been consistently smaller than the acceptable quality level (AQL) specified. When the process average over at least 10 lots has been very much smaller than the AQL, some consumers resort to skip-lot procedures (see ISO 2859-3). This can be even more economical than the reduced inspection described in ISO 2859-1.

In some instances, particularly when routine or noncritical items are involved, some consumers may feel safe in resorting to the practice of inspecting small samples of the product and, provided there are zero nonconforming items, accepting the lot. For example, with a sample size of eight this is equivalent to the small lot single sampling plans with an AQL of 1,5 % normal, or 0,65 % reduced inspection. See Tables II-A and II-C in ISO 2859-1:1989.

Conversely, in ISO 2859-1, when two out of five successive lots fail acceptance, normal inspection is discontinued, and tightened inspection is instituted. Once tightened inspection has been instituted, normal inspection is not restored until five successive lots have been accepted on tightened inspection. This requirement is intentionally severe, because evidence of unacceptable quality has been found. The producer then forfeits the right to the benefit of the doubt. If, while operating on tightened inspection, the cumulative number of lots not accepted on original tightened inspection reaches five, inspection by sampling should be discontinued until there is evidence that corrective action has been taken and has been effective. See **2.11**.

### 2.2.2 Ad hoc sampling

*Ad hoc* sampling is not to be recommended as it will lead to uncalculated risks that may be unjustifiably high; furthermore, there is no formal basis for either the acceptance or non-acceptance of the lot.

**2.2.3 100 % inspection**

100 % inspection can be a formidable task unless the 100 % inspection is performed with automatic test equipment. In addition, it is not always successful, particularly when a large number of items have one or more characteristics that are marginal dimensionally, in appearance or in performance (close to or concentrated about a tolerance or limit of appearance or performance). Under these conditions, sorting by manual or automatic methods is likely to classify some conforming items as nonconforming, and vice versa. In addition, 100 % testing by manual, visual or automatic methods can be unsatisfactory. It can sometimes degenerate into superficial 100 % inspection when, in fact, sufficient money, time and staff are not available. 100 % inspection is not viable if the inspection method necessitates destructive testing.

It has to be understood, however, that 100 % inspection may form a necessary part of the inspection process for both the consumer and the producer. There are situations in which it cannot be avoided, for example when inspecting for critical nonconformities, as will be clear from a study of this part of ISO 2859. Some types of nonconformity are so important that every item has to be examined when tests are nondestructive. When the tests are destructive, some risk has to be accepted. (See **2.15**).

**2.2.4 Other "sampling" practices**

Various sampling systems exist but only those available as International Standards will be considered in detail in this part of ISO 2859. This should not be taken as meaning that the others are unimportant; it is merely that the main purpose of this part of ISO 2859 is to help people to use parts 1, 2 or 3 of ISO 2859 or ISO 8422.

In many instances, consumers do not perform any regular sampling but rely on their experience and supporting evidence that the producer is maintaining statistical control of his production process and is forthright in his evaluation of what is being shipped.

If, in a particular situation, information is available of the true costs of the mistaken non-acceptance of good articles and the acceptance of bad ones, and if from long experience it is known how often lots of any given quality are presented, this may be one of the occasions when compromise is not desirable. It may be possible to calculate a more efficient scheme on the basis of the economic information available.

## 2.3 Choosing between attributes and variables inspection

The attributes method of inspection consists of examining an item, or characteristics of an item, and classifying the item as "conforming" or "nonconforming". The action to be taken is decided by counting the number of nonconforming items or the number of nonconformities found in a random sample.

The variables method starts with selecting a sample of a number of items and measuring dimensions or characteristics so that information is available not only on whether a dimension, for example, is within certain limits but on the actual value of the dimension. The decision whether or not to accept a lot is made on the basis of calculations of the average and the variability of the measurements in accordance with the procedures of ISO 3951 or ISO 8423.

Provided certain assumptions are true, the variables method has the advantage of requiring a smaller sample size than the attributes method to attain a given degree of protection against incorrect decisions. Also it provides more information as to whether quality is being adversely affected by process mean, process variability or both. The attributes method has the advantage that it is more robust (not subject to assumptions of distributional shape) and that it is simpler to use. The larger sample sizes and the increased costs associated with using attribute sampling methods may be justifiable for these reasons.

It should be noted that go, no-go gauging is faster and requires less skill than measurement.

Both methods have advantages and typical fields of application.

Although occasionally reference is made to ISO 3951 and ISO 8423 in subsequent clauses, variables schemes, as such, are not considered further in this part of ISO 2859. ISO 3951 and ISO 8423 include guidance on their use.

## 2.4 Lot inspection

**2.4.1 Lot**

For the purposes of ISO 2859, ISO 8422, ISO 3951 and ISO 8423, items are offered for acceptance in groups, not on a single item basis. Each group of items is called a lot.

Each lot should, as far as is practicable, consist of items manufactured under essentially the same conditions during one time period. This is of the utmost importance if the acceptable quality level concept is adopted and there are a series of lots to be delivered.

If two or more sources of supply are mixed, the presence of a large number of nonconforming items from one of the sources can result in non-acceptance of the product from all the sources. Conversely, product of marginal quality from one source can be masked by mixing with product from sources of excellent quality.

From each lot a sample is drawn and inspected. Under attributes inspection, each lot is classified as acceptable or unacceptable on the basis of the number of nonconforming items or nonconformities found in the sample. Each successive lot is therefore dealt with as a more or less independent unit (although the rules for sentencing may sometimes vary according to the results from preceding lots).

For single lots offered in isolation, see **2.5.2**.

### 2.4.2 Lot size

The responsible authority (see **3.10** in ISO 2859-1:1989) has the right to specify what is to be the lot size, but it clearly makes sense that, where possible, this should be done in consultation with the producer, so that a quantity that is mutually convenient may be chosen. Certainly, specifying the lot size (and other parameters of the sampling plan) should never be done in ignorance of the production process. It is not essential that an inviolable quantity should be chosen. Sometimes variation may be allowed, although it will nearly always be desirable that upper and lower limits of the lot size should be specified.

From the sampling inspection point of view, there is an advantage in large lots, as from a large lot it is economical to take a large sample, thereby achieving better discrimination between good lots and bad ones. With large lots, the required sample size is a smaller proportion of the lot than with small lots for the same AQL.

This “large lot” policy should not be overdone, however. If making up a large lot necessitates putting together smaller lots that could have remained separate, then a large lot is advantageous only if the smaller lots are of a similar quality. If there is likely to be any substantial difference between the qualities of the smaller lots, then it is much better to keep them separate. For this reason, lots should consist of items of product produced under essentially the same conditions.

Examples of the formation of lots are given in **3.4**. More information on the lot size/sample size relation is given in ISO/TR 8550.

## 2.5 Sequence or isolated lot inspection

### 2.5.1 Lot-by-lot inspection

Lot-by-lot inspection is the inspection of product submitted in a series of lots.

If a sequence of lots is to be offered for acceptance at the time of production, the inspection results from the preceding lots can be avaible before the later lots are made. It is therefore possible that the inspection performed can beneficially influence the quality of subsequent production. The lots should be submitted and inspected in the same sequence as they are manufactured and inspection should be made promptly. Information obtained from a lot may indicate that the process appears to have deteriorated. The information obtained from several lots in sequence can be used to invoke a switching procedure which requires the use of a more rigorous sampling procedure in the event that the process deteriorates. This is important because, in the long run, it provides the best protection a consumer has against poor quality. If the quality remains poor, then under the more rigorous sampling practice more lots will be returned to the vendor for sorting. This tighter sampling increases the producer’s risk of having an acceptable lot judged unacceptable. The identification of possible deterioration in product quality is a signal to initiate corrective action.

If the quality is very much better than that agreed upon, the consumer may, with the permission of the responsible authority, elect to adopt reduced or skip-lot sampling.

ISO 2859-1, ISO 2859-3, ISO 8422, ISO 3951 and ISO 8423 are designed principally for use with a sequence of lots.

### 2.5.2 Isolated lot inspection

Inspection may sometimes be performed on an isolated lot, just a few isolated lots, or on stored lots at a time when production has been finished. Under these circumstances, there is insufficient opportunity for the switching rules to be invoked and hence to influence the quality to be offered.

If a single lot is to be delivered, then it is helpful to know whether the lot is one of many similar lots delivered to other consumers and consists of material from a controlled process or whether it is a mixed lot containing items from different processes and different times. (See also ISO/TR 8550 and ISO 2859-2).

Whereas ISO 2859-1 and ISO 2859-3 call for establishing the AQL value and the inspection level in advance, ISO 2859-2 requires the establishment of the limiting quality (LQ). In order to provide appropriate producer and consumer protection when lots are sampled under the limiting-quality procedure, information is needed as to whether the lot came from a continuing series of acceptable lots, or is a mixed lot, consisting of product made on different production lines and/or different dates.

The tables in ISO 2859-2 are designed principally for use with isolated lots.

## 2.6 Acceptable quality level (AQL)

### 2.6.1 Description

The acceptable quality level is used as an indexing device in the tables of ISO 2859-1, in ISO 3951, and in some of the tables of ISO 8422 and ISO 8423.

When using these AQL-indexed sampling plans, inspection lots taken from a process whose quality is equal to or better than the AQL will be accepted most of the time.

When a continuing series of lots is considered, the AQL is a quality level which for the purposes of sampling inspection is the limit of a satisfactory process average.

The AQL is a chosen borderline between what will be considered acceptable as a process average, and what will not. As such, it in no way describes a sampling plan, but is a requirement of what the production should be like, and is a useful quantity to consider in defining a tolerable process.

The fact that an AQL is specified should not be taken to imply that a percentage of nonconforming items up to the specified value is wanted, or is completely acceptable, it is always better to have no nonconforming items than any percentage whatever, and the more the percentage can be reduced below the AQL the better. This reduction improves the probability that each lot is accepted.

### 2.6.2 Setting an AQL

In setting an AQL, it has to be remembered that the AQL provides an indication of the quality that is required in production. The producer is being asked to produce lots of an average quality better than the AQL. On the one hand, this quality has to be reasonably attainable, whilst on the other hand it has to be a reasonable quality from the consumer's point of view. Frequently this will mean a compromise between the quality the consumer would like and the quality he can afford, for the tighter the requirement the more difficult it may be for the production to meet it, and the more expensive may be the inspection to ensure that it is met.

A properly designed and controlled process may be capable of producing product with a smaller percentage nonconforming than the AQL value. When a better process average is obtainable from a process, the cost of production plus the cost of inspection will be lower for the better quality.

The primary consideration has to be the consumer's requirement, but it is necessary to make sure that the consumer is being realistic and is not demanding something tighter than is really needed. It is necessary to take into account how the items in question are to be used and the consequences of a failure. If the items are to be available in large numbers and the failure is simply a failure to assemble so that the nonconforming item can be put aside and another used in its place, a relatively generous AQL may be tolerable. If, on the other hand, a failure is going to cause a failure to function of an expensive and important piece of equipment at a time and place where a replacement of the nonconforming item cannot be made, a tighter AQL will be required.

More information and guidance on setting an AQL is given in **3.9** and in ISO/TR 8550.

## 2.7 Process average

The process average is the average quality submitted over a series of lots, resubmitted lots being excluded.

It is particularly important to realize that, in contrast to the AQL, the AOQL (see **2.12**) or the LQ (see **2.8**), the process average is not something that can be calculated or chosen, or is a property of a particular sampling plan. The process average relates to what is actually produced, irrespective of what inspection is performed.

Generally, the estimation of a process average is not an essential part of a sampling scheme. However, the process average is important in its own right. Both the inspector and the producer are interested not only in the lot-by-lot decisions but also in the long-term picture of the quality of production.

It is, therefore, desirable to keep a record of the overall estimated process average being achieved because this gives a useful measure of quality and is also invaluable information for those who have to decide what sampling plans should be adopted when similar products are being designed and made in the future.

Special rules need to be observed where the sampling is of the double or multiple form. Only the results of the first sample in double and multiple sampling should be used to estimate the process average.

Occasionally a recommendation is made that abnormal results should be excluded. This is a dangerous practice that should be used very sparingly, if at all. The only time this practice may safely be adopted is if the abnormal results are known to be due to a specific cause which is known to have been eliminated. Even then it is good practice to quote figures which include and which exclude these abnormal results to indicate that these nonconformities did exist.

Separate process averages have to be estimated in the case of multiple characteristics or multiple AQL classes.

## 2.8 Limiting quality (LQ)

Limiting quality is an indexing device used in ISO 2859-2. When a lot is considered in isolation, LQ is a quality level in percent nonconforming (or nonconformities per 100 items) which for the purposes of sampling inspection is limited to a low probability of acceptance. This small probability of acceptance is called the "consumer's risk".

Specifying a limiting quality is in fact the specification of a quality that is not wanted! To have lots regularly prove to be acceptable, the fraction of nonconforming items has to be much smaller than the LQ (usually less than a quarter of the LQ).

ISO 2859-2 provides procedures for the application of LQ sampling plans. These sampling plans and tables are for the most part consistent with the sampling plans used in ISO 2859-1.

Limiting quality plans are used primarily for isolated lots. When the product is in manufacture and there are a series of lots being produced, the procedures of ISO 2859-1 are more appropriate.

## 2.9 Normal and tightened inspection

An AQL, it will be remembered, is the borderline in the quality scale between the good and the bad when a sequence of lots is inspected. When the AQL has been specified for any particular product, the ideal would be to have a system whereby lots could be always accepted when their quality was better than the AQL and always not accepted when worse than the AQL. This ideal is not attainable with any sampling plan.

To meet the requirements of both the producer and the consumer, some compromise is needed, and the device adopted in ISO 2859-1 and ISO 8422 is to join normal inspection with tightened inspection; i.e. two sampling plans are specified for any given situation, together with rules for determining when to switch from one to the other and when to switch back again.

Normal inspection is designed to protect the producer against having a high proportion of lots not accepted when quality is better than the AQL. In fact, the producer is being given the benefit of any doubt that arises due to sampling variability.

But the consumer needs protection too, and this is achieved by arranging that the producer is not given the benefit of the doubt blindly and invariably, but only for as long as he proves worthy of it. If at any time the sampling results show that his process average is probably worse than the AQL, he forfeits his right to the benefit of the doubt (that is, his right to normal inspection), and tightened inspection is instituted to protect the consumer.

Further details with examples are contained in **3.11** and **3.12**.

## 2.10 Reduced inspection

Sometimes there is evidence that the product quality is consistently better than the AQL. Where this happens and there is reason to believe that good production will continue, sampling inspection no longer serves the purpose of segregating the good lots from the bad ones. However, inspection cannot be dispensed with altogether, as a warning is needed if the production quality worsens.

In these circumstances, considerable savings can be made if so desired by using the reduced-inspection sampling plans described in ISO 2859-1 or the skip-lot sampling plans described in ISO 2859-3. The special rules, for allowing the use of these plans, if permitted by the responsible authority, are described in ISO 2859-1, ISO 2859-3 and also in Section 3 of this part of ISO 2859.

Reduced inspection is further discussed, with examples, in **3.15**.

## 2.11 Switching rules

Subclause **2.9** introduced normal inspection and tightened inspection and their purpose. This subclause discusses the switching rules by means of which the decision is taken to change from normal to tightened inspection or back again when using ISO 2859-1.

If the actual value of the quality being offered by the producer were known, the knowledge would be used to sentence the lots instead of submitting them to acceptance inspection. As the actual quality is never known, the best that can be done is to use the knowledge that is available, i.e. the sampling inspection results themselves.

As normal inspection is designed to accept nearly all the lots offered, provided that the quality is at least as good as the AQL, it follows that if a high proportion of lots is not being accepted, the quality cannot be as good as the AQL.The question is: “What proportion of non-acceptance is high enough to be convincing?” A rule is required that will give reasonably quick reaction if quality becomes worse than the AQL, while having a low probability of calling, in error, for tightened inspection when the quality is really better than the AQL.

The rule used is that tightened inspection has to be used for the following lots as soon as two out of any five or fewer successive lots on original inspection have not been accepted. The qualification “on original inspection” means that if lots are not accepted but resubmitted after rectification, these resubmitted lots are not counted for switching-rule purposes.

Once tightened inspection has been instituted, it remains in force for every lot until five successive lots have been accepted on tightened inspection, then normal inspection is restored. This requirement is quite a severe one, as acceptance on tightened inspection is more difficult than on normal inspection, but once there is evidence that quality worse than the AQL has been produced, the producer’s right to the benefit of the doubt cannot be restored until it is safe to do so.

There is one further safeguard for the consumer. This is the rule that acceptance inspection should be discontinued, pending action to improve the quality, if the cumulative number of lots not accepted in a sequence of consecutive lots on original tightened inspection reaches five. This is a most important principle; if the quality is bad, action is needed, and the inspector has to be entitled to refuse to inspect any further lots until he has evidence that suitable action has been taken.

An example is given in **3.13**.

## 2.12 Average outgoing quality (AOQ) and its limit (AOQL)

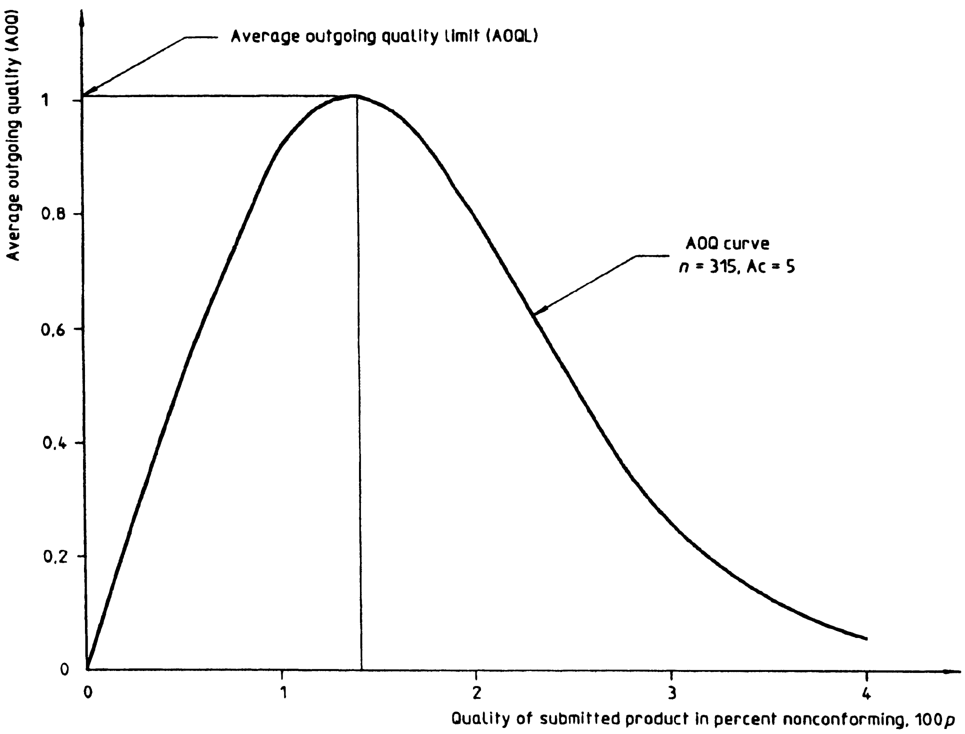
As with the AQL concept, the concept of average outgoing quality and its limit is only meaningful when a long sequence of lots is submitted to a defined system of sampling inspection, e.g. in accordance with the provisions of ISO 2859-1. When the number of nonconforming items in the sample is equal to or less than the acceptance number, the lot will be accepted. Conversely, when the number of nonconforming items in the sample is equal to or greater than the rejection number, the lot will not be accepted. When the supply (or source) process operates at a process average close to the specified AQL, most of the lots will be accepted. Provided that process quality is constant and non-accepted lots are discarded rather than rectified, the effect of sampling on the quality is nil.

In some instances, particularly when the transfer is between departments rather than companies, the result of a lot failing to pass sampling inspection is that the lot is 100 % inspected and the nonconforming items removed (and perhaps replaced with conforming items). This is termed “rectifying inspection”.

When lots are submitted to rectifying inspection, the lot is either accepted with no further inspection or, when the sample indicates non-acceptance, all the items in the lot are inspected and nonconforming items discarded or replaced by conforming items. In the first case, the outgoing quality is, for practical purposes, the same as the incoming quality; in the second case, all items conform to the specification. Even though the incoming quality may be constant at $p$ (fraction nonconforming) the process average, the outgoing quality will vary from lot to lot, taking either the value $p$ or zero depending on whether the lot is accepted on the sample result or is subjected to rectifying inspection. It is possible, however, to think of the average of these outgoing qualities over a long run in which the incoming quality was constant at $p$. This average of the outgoing quality will clearly not be greater than $p$ and, where a large proportion of lots is completely inspected, it can be very much less.

The term "average outgoing quality" can be thought of as the average percent nonconforming over many lots from a process continually delivering product of quality $p$. Each lot is examined and sentenced by the same sampling plan which has a probability $P_a$ of accepting the lot. Those lots which are not accepted by the sampling plan are cleared (theoretically) of all nonconforming items. The result, on the average, is that, after inspection, $100(1 - P_a)$ % lots are 100 % conforming and the $100P_a$ % lots, which have been inspected by sampling alone, contain a percentage $100p$ of nonconforming items (minus a few removed during sampling). The average outgoing quality, in percent nonconforming, will be approximately $100(P_a \times p)$ %. The approximation is good if the lot size ($N$) is at least 10 times the sample size ($n$).

Performing this calculation for varying values of $p$, each of which has a different probability of acceptance, will result in an average outgoing quality curve as in Figure 1. It is clear from this figure that outgoing quality can be good either because incoming quality was good or because the lot was completely inspected. It is further clear that there is an intermediate incoming quality ($p$) for which the average outgoing quality achieves a maximum value. This maximum value is the AOQL. It is not a limit on the outgoing quality from any one particular lot nor is it a limit on the actual outgoing quality averaged over a short sequence of lots. In a long sequence of lots, however, the actual outgoing quality average over that sequence will not be significantly different from this AOQL. If the input quality has varied from the incoming quality ($p$), then the actual quality may be very much better than the AOQL. It is therefore good practice to estimate the actual average quality directly rather than to rely on the AOQL as an upper boundary.




$n$ = sample size

Ac = acceptance number

**Figure 1 — AOQ and its limit AOQL**

## 2.13 Item; unit of product

In using attributes inspection, it is necessary to count such things as lot size, sample size, number of non-conforming items, etc. This counting is done in terms of the unit of product. The term "item" has been adopted by ISO for the unit of product to avoid confusion with units such as centimetres, grams, etc. Usually, the item will be a single article (of the type being inspected) and when this is so, the word "article" may be used if desired instead of item.

EXAMPLE 1

Vitrified clay pipes, with a nominal length of 3 m and nominal diameter 150 mm, are due to be delivered as a lot of 250 pipes. Before delivery, a random sample of eight pipes is drawn from the lot and these eight pipes are tested for crushing strength, bending moment resistance and impermeability. The lot is delivered if, and only if, there are at least seven pipes conforming to the specified tolerances of all three of these characteristics. Here the item is the individual pipe.

NOTE 2 The reason for introducing the term "item" is that it is sometimes desired to perform sampling inspection of a product which does not consist of individual articles, or in which the basic entity being inspected consists of a number of articles.

EXAMPLE 2

A sand-cement mix (10 000 kg) is packaged in 10 kg bags. The lot is to be inspected for compliance with a specification which defines the maximum grain sizes of the sand and the cement, the proportion of sand to cement, and the weight of each bag.

The AQL is specified. The specification defines as conforming those bags where less than $x$ % of the sand grains exceed a size $a$ and less than $y$ % of cement grains exceed a size $b$, etc.

The lot contains 1 000 units (bags). Specifying an inspection level S4 and an AQL of 2,5 % provides the basis for determining the appropriate sample size (20 bags) and the acceptance number (1). (See Tables I and X-F-2 of ISO 2859-1:1989).

The item is a 10 kg bag. The bulk material is granular and could not be otherwise treated as any number of individual items.

EXAMPLE 3

A piece of electronic equipment contains in its circuit two similar transistors and it is important for correct functioning that the electrical characteristics of these transistors should be closely matched. In this case, the item to be inspected is defined as a matched pair of transistors. 500 matched pairs, a total of 1 000 transistors, would then give 500 items. The lot size is 500. If the sample size necessary were, say, 50, this would mean 50 pairs, i.e. a total of 100 transistors. It would, of course, be necessary in this situation for the pairs to be defined before the sample was drawn, and the pairs would have to be kept intact right up to the time of use. In the case of electronic transistors consisting of two similar structures within the same package or on a single substrate, the entire article would be one item, although it would be necessary to test each transistor individually.

EXAMPLE 4

50 000 pieces consisting of 25 000 cans and 25 000 covers, which are specially formed mating parts, are produced and shipped to an assembly plant on a daily basis. The process of manufacture and use is such that mating parts are not separated but have to be assembled as a pair.

One of the most important checks is: Do parts mate properly? The item for inspection is a pair, one can and one cover identified as one pair.

## 2.14 Nonconformity and nonconforming item

### 2.14.1 Failure to conform

For the purposes of ISO 2859 and ISO 8422, any failure to conform with a specified characteristic, dimension, attribute or performance requirement represents a nonconformity. A nonconforming item may have one or more nonconformities.

For example, suppose that a ball-point pen fails to write. The failure to write is a nonconformity; the pen is nonconforming. The same pen could also have failed to meet its specification in a number of other ways, e.g. colour, dimensions, etc. Although it exhibited several nonconformities, it would be counted as one nonconforming item.

The qualification "nonconformity" does not necessarily imply that the unit of product cannot be used for the purpose intended. For example, a brick with one of its dimensions outside the prescribed tolerance interval, though nonconforming, can still be used for building.

The distinction between nonconformity and nonconforming item is of no importance if the items have no more than one nonconformity, but becomes essential when multiple nonconformities can occur.

The quality of a given quantity of product may be expressed either as percent nonconforming or as the number of nonconformities per 100 items, but these are not usually interchangeable.

Sampling plans are available for either the percent nonconforming or the number of nonconformities per 100 items.

EXAMPLE 5

In counting pinholes in metal foil, the number of pinholes per square metre may be of interest. Here all the pinholes in each square metre (item) examined would be counted and the quality would be expressed in pinholes/100 $m^2$.

EXAMPLE 6

Suppose there is a lot of 500 articles. Of these, 480 conform and are acceptable, 15 have one nonconformity each, 4 have two nonconformities each, and 1 has three nonconformities.

The lot percent nonconforming is given by the formula:

Percent nonconforming

$$= \frac{\text{No. of nonconforming items}}{\text{total No. of items}} \times 100$$

= (20/500) × 100 = 4

That is, the lot is 4 % nonconforming.

The number of nonconformities per 100 items in the lot is given by the formula:

Nonconformities per 100 items

$$= \frac{\text{No. of nonconformities}}{\text{total No. of items}} \times 100$$

= (26/500) × 100 = 5,2

That is, the lot has 5,2 nonconformities per 100 items.

Whether percent nonconforming or nonconformities per 100 items is to be used is a matter for individual consideration in each particular case. The important thing is that it has to be considered, specified and agreed upon beforehand, not left until a sample has been inspected and then considered.

Factors to be taken into account in deciding whether to use percent nonconforming or nonconformities per 100 items are as follows.

a) Inspection for percent nonconforming assumes that if an item contains one or more nonconformities, the item is nonconforming and is not acceptable. It also presupposes that the number of different ways in which an item can be nonconforming is limited and known, e.g. there are only five ways in which each particular item could be nonconforming [see also b)].

Under the conditions of inspection for percent nonconforming, a record should be kept of all nonconformities found in each of the nonacceptable items, so that corrective action can be taken for each type of nonconformity. No differentiation need be made in the count. An item with one nonconformity or an item with several is counted as a nonconforming item.

b) Inspection for nonconformities per 100 items counts each nonconformity found. Three nonconformities found in one item count as three, and are given the same weight as three items each with one nonconformity.

A special case arises when a nonconformity can occur an unknown and almost unlimited number of times in items, for example, surface blemishes or pinholes can occur in any number and it is not known how many times they do not occur, so percent nonconforming for this feature is meaningless. In such cases, nonconformities per 100 items should be used (see example 5).

NOTE 3 Percent nonconforming implies a binomial distribution. For nonconformities per 100 items, a Poisson distribution is appropriate. See **2.19**. for information on the operating characteristic curves of sampling plans.

c) Two properties will be dependent if nonconformities in an item arise, in part or wholly, through some common cause, or if one property affects the other. Detailed knowledge of the production process is thus needed to decide that properties are independent. In mathematical terms, if two characteristics (say, length and diameter) are independent, it means that if all the units produced were taken and sorted into two groups according to whether the length was nonconforming or not, then the percentage nonconforming for diameter would be found to be essentially the same in each of these two groups; or, alternatively, if they were sorted into two groups according to whether the diameter was nonconforming or not, then the percentage nonconforming for length would be essentially the same in the two groups. It can be shown mathematically that these two procedures are equivalent.

If two nonconformities are not independent, then they are said to be related or dependent. It should be agreed that the occurrence of both in one item is to count as only one nonconformity, not as two. Occasionally the correlation between two related nonconformities is low. Under these conditions, the two may be considered to be independent. Inspection for percent nonconforming avoids this difficulty.

d) If the percentage of nonconformities in the lot is less than 2,5 %, then the probability distributions of nonconforming items and nonconformities will be almost identical. In the range 2,5 % to 10 %, some difference will be apparent, a nonconformities per 100 items plan being rather more severe than the equivalent percent nonconforming plan.

e) At an inspection station and where admissible, it may be simpler and better practice to use one method rather than to change frequently from one method to the other, for example nonconforming items rather than nonconformities per 100 items.

f) From the point of view of keeping records that will be useful for improving quality, nonconformities per 100 items is preferable as the records will then automatically contain information on all nonconformities, whereas some nonconformities may escape the record if the percent nonconforming approach is adopted.

**2.14.2 Nomenclature**

The discussion in the remainder of this part of ISO 2859 will be in terms of inspection for nonconforming items. When appropriate, it may be read in terms of inspection for nonconformities, by replacing "nonconforming items" by "nonconformities", and "percent nonconforming" by "nonconformities per 100 items".

**2.14.3 Classification of nonconformities**

The discussion so far has assumed that, if an article can be nonconforming in more than one way, the different possible nonconformities are all of equal importance. It is then possible to sentence by counting the nonconforming items. For example, if there are three dimensions to be checked and, in a sample, three articles are nonconforming in the first dimension alone, three articles in the second dimension alone, one article in the third dimension alone, and one article in both the first and second dimensions, this gives a total of eight nonconforming items, which is the number to compare with the acceptance and rejection numbers.

The procedure of adding nonconforming items of different types is reasonable only if the nonconformities are of equal, or nearly equal, importance. Where this is not so, it is necessary to classify the possible nonconformities into groups so that nonconformities in different groups are of different orders of importance but all nonconformities within a group are of approximately the same order of importance. Different AQLs are then used for the different groups.

For many purposes, two groups are sufficient, namely major nonconformities of class A which are of greatest concern and nonconformities of class B which are of the next greatest concern. Sometimes it is necessary to introduce further classes or sub-classes within these classes. The most important class of all contains the critical nonconformities which render the articles hazardous, potentially hazardous, or adversely affect usage.

Critical nonconformities are a special case and are discussed in more detail in **2.15**. For the moment, the discussion will be restricted to the major and minor classes. It should be realized that these classes refer to the relative importance of different nonconformities within any given product, and as products themselves vary in importance, the classes do not correspond to any absolute standards. There is, therefore, no particular AQL that normally goes with any class.

The classifying of nonconformities should be properly done. It is clear that care has to be taken not to "under-classify" (for example, to classify as a class B nonconformity a feature that should be in class A), as this will lead to the allowance of more nonconformities of this class in the plan for the feature concerned than is really required. It is often not realized, however, that it is also very important not to "over-classify".

When the system of classification of nonconformities is adopted, it is necessary to allocate a different AQL to each class to ensure that the more important, class A, nonconformities are more tightly controlled than the class B nonconformities.

If an article has more than one nonconformity and the nonconformities come within different classes, it counts as a nonconforming item of the more serious class. (If, however, inspection is in terms of nonconformities rather than in terms of nonconforming items, each nonconformity in the sample is counted in its appropriate class.)

Further information and examples of classification are given in **3.3**.

## 2.15 Critical nonconformities

Critical nonconformities, by definition, present a hazard and/or adversely affect usage or safety. These nonconformities form a special category. It is impossible to choose any value of percent nonconforming for these nonconformities and say: "... this percentage of critical nonconformities is tolerable".

Where non-destructive inspection is involved, the solution generally adopted is to require that critical characteristics are to be inspected using a sample size equal to the lot size and an acceptance number of zero. This is 100 % inspection, but it should be noted that it is not the traditional 100 % sorting. There is no attempt here to sort the articles into the good and the bad but an attempt to check that there are no bad ones. If a critical nonconformity is found, this does not merely mean that it is put into a different box and the inspection continues; it means that the whole lot is not accepted [although non-acceptance does not necessarily mean scrapping (see **2.17**)]. Whenever possible, it should also mean that production is stopped while a thorough investigation takes place to attempt to discover how the nonconformity arose and to devise methods to prevent another occurrence. The reason for this procedure is to try to prevent the production of items with serious nonconformities and to avoid giving the producer the impression that it will not matter too much if some are produced as the inspector will sort them out. Even the best inspector may occasionally fail to notice a nonconformity, so it is only by preventing critical nonconformities from being made that it can be ensured that none will get through to the consumer.

If it is ever thought that any particular critical nonconformity does not warrant this procedure, then serious consideration should be given to having it reclassified as a major nonconformity. Critical nonconformities really have to be critical; then no amount of effort is too great.

Where the only possible inspection for critical nonconformities is destructive, the search for ways of preventing them from ever being made at all is even more important. In this case, we cannot have a sample which is 100 % of the lot, and it is necessary to decide what sample should be taken. This can be done using a simple formula relating

a) the number of nonconformities/nonconforming items for which, if they were present, we would wish to be almost certain of finding at least one nonconformity/nonconforming item in the sample;

b) the lot size;

c) the sample size; and

d) the risk we are prepared to take of failing to find a nonconformity/nonconforming item.

Obtain the sample size ($n$) from the following formula and then round up to the nearest integer[1)]. The lot is acceptable if no critical nonconformities are found in the sample:

$$n = (N - d/2)(1 - \beta^{1/(d+1)}) \quad \text{...(2.1)}$$

where

$N$ is the lot size;

$\beta$ is the specified probability of failing to find at least one critical nonconformity;

$d$ is the maximum number of critically nonconforming items "allowed" in the lot.

If $p$ is the maximum fraction nonconforming specified for the lot, then

$d = Np$ rounded down to the nearest integer[2)]

EXAMPLE 7

Suppose there is a lot of 3 454 items. A probability, $\beta$, of 0,001 and a maximum percentage of 0,2 % critically nonconforming items are specified.

Then

$p = 0{,}2/100 = 0{,}002$ and

$Np = 3\ 454 \times 0{,}002 = 6{,}908$

which is rounded down to give $d = 6$.

Thus

$$(N - d/2)(1 - \beta^{1/(d+1)}) = (3\ 454 - 3)(1 - 0{,}001^{1/7})$$

$$= 3\ 451 \times 0{,}627\ 24 = 2\ 164{,}61$$

which is rounded up to give $n = 2\ 165$.

The sampling plan is:

| | |
|---|---|
| sample size | n = 2 165 |
| acceptance number | Ac = 0 nonconforming items |
| rejection number | Re = 1 nonconforming item |

1) This approximation is accurate enough for most practical purposes in acceptance sampling. In rare cases it will give a result which is one unit larger than necessary.

2) Only small values of percent nonconforming should be considered tolerable, as the non conformities are critical.

EXAMPLE 8

To find the lot size, $N$, needed to yield a specified number of items, $L$, after destruction of the sample of $n$ items under test, assuming no nonconforming items are found, then for given values of the probability $\beta$ and the number of nonconforming items in the lot, the lot size is (rounded up):

$$N = (L - d/2)/\beta^{1/(d+1)} + d/2 \qquad \text{...(2.2)}$$

Now, suppose that 1 500 items are required after testing the sample, using $\beta = 0{,}001$ and $d = 6$ as in example 7. Then $L$ is 1 500 and the lot size is

$$(1\,500 - 6/2)/0{,}001^{1/7} + 6/2$$

$$= 1497/0{,}372\,76 + 3 = 4\,018{,}99$$

which is rounded up to give $N = 4\,019$.

It follows that

$$n = N - L = 4\,019 - 1\,500 = 2\,519$$

This value of $n$ is also obtained using equation (2.1) with a lot size of 4 019.

If the initial calculation yields an unacceptable sample or lot size, then the risk (probability) and/or the possible number of nonconformities/nonconforming items in the lot need(s) to be reassessed and new criteria established.

An alternative plan for critical nonconformities, where the critical characteristic is something that can be measured rather than a pure attribute, is to sample with a safety margin. Thus, if the minimum allowable breaking load for some component were 2 000 kg, it might be possible, instead of saying that the limit was 2 000 kg and the nonconformity was critical, to say that the limit was 2 500 kg and the nonconformity was major. Just where the limits should be set and what plan is allowable depend upon some past knowledge of the amount of variability observed in the strength of the components in question. When this approach is possible, it can give much more satisfactory results for all concerned than 100 % inspection. There is, in this case, the possibility of sampling by variables (ISO 3951) which will allow over-stress testing and will yield information on the average and the variability of the characteristics.

## 2.16 Curtailment of inspection

As inspection of the items in the sample proceeds, the action to be taken may become more and more evident; when all the items in the sample have been tested, the decision can be made according to the criterion of the sampling plan used. It may happen that this decision can be predicted with certainty at some earlier stage because there are already sufficient conforming items to force acceptance whatever later items may show, or because there are already sufficient nonconforming items to force nonacceptance. For example, if the sample size is 80 and the acceptance number is 10, then finding 11 nonconforming items in the first 20 items tested will force a decision not to accept the lot even if the remaining items were all good. If inspection is stopped as soon as the final decision can be predicted with certainty then the inspection is said to be curtailed. Note that inspection cannot be stopped before the final decision is certain without invalidating the operating characteristics of the plan. There are obvious savings in inspection costs to be gained from curtailed inspection. However, there are less obvious disadvantages arising from this practice.

Two of the purposes of sampling inspection of a sequence of lots are to obtain information on the nonconformities in the product and to estimate the process average for that sequence of lots. When the testing is complete for each sample (not curtailed), the proportion nonconforming in the total of all the samples is an unbiased estimate of the process average. If the inspection is curtailed, this simple procedure will no longer give an unbiased estimate of the process quality and the samples cannot be treated as if the sample sizes achieved in curtailment were those intended in uncurtailed inspection. The loss of information from the items not tested is another of the disadvantages associated with curtailment. A third disadvantage can be the extra administrative effort needed to make the individual test results available in sequence in order to effect curtailment.

Double, multiple and sequential sampling plans may be used to save on the number of items tested. The respective average savings in testing cost can be as much as 3/8, 1/2 or 5/8 of the cost of single sampling. Curtailed sampling cannot match these average savings when the input quality is good, as the major saving from curtailed inspection occurs when the lot is not accepted. There is, therefore, no justification for curtailed inspection in single sampling inspection in preference to the alternative double or multiple sampling plans that use a fixed smaller size sample as a first stage in the decision procedure. With double or multiple sampling, the process average may be estimated by the percent nonconforming in the first sample from each lot or by the overall percent nonconforming in a number of first samples.

When double or multiple sampling plans are being used, it is common practice to curtail sampling in the second or later samples because these data are not utilized for estimation of the process average.

## 2.17 Disposal of unacceptable lots

When a lot fails the acceptance sampling criteria, it is not accepted and the consumer has several choices which are dependent on commercial arrangements.

The lot may require sorting by 100 % inspection, reprocessing or scrapping.

If the lot is returned to the producer for such reprocessing or other disposal, it is essential that it be identified as a re-submitted lot if it is returned to the consumer. It should be noted that if the lots are offered for acceptance sampling inspection often enough, even if there is only a small chance of acceptance each time, eventually they will be accepted.

Any producer who did this would be harming himself, for if the lot were offered as a new lot several times it would appear that several bad lots had come up in succession. This would lead to the belief that quality was worse than might be the case. This would lead to a change to a tighter sampling plan, followed by the discontinuation of inspection pending quality improvement.

Non-acceptance does not necessarily mean scrapping. According to the circumstances of the particular case, the lot may be scrapped, 100 % inspected with either rectification or replacement of nonconforming items found, accepted at a reduced price, or accepted for another purpose, use or application.

If 100 % inspection with rework, rectification or replacement of nonconforming items is allowed, the lot will eventually be re-submitted for inspection. The inspector needs to know that this is a re-submitted lot so that he may give particular attention to the features for which it was not accepted. The results of inspecting re-submitted lots should be recorded separately from the records of lots offered for original inspection so as not to cause confusion in any calculations of product quality. The results should not be used in determining whether the switching rules should be invoked.

Whether to inspect all classes of features of a resubmitted lot, or only the classes of features that caused non-acceptance, is largely an administrative decision depending upon the conditions of the particular case. In the case of rework or reprocessing, consideration has to be given to the possible adverse effects such action may have on other features.

## 2.18 Single sampling

A single sampling plan is described by three numbers: the sample size, the acceptance number and the rejection number. The plan is operated by drawing from the lot at random (see **2.25**) the number of items of product required to make up the sample size. The items of product drawn are then known as “sample items”, and collectively as “a sample”.

The sample is inspected and the number of nonconforming items discovered is counted. If the number of nonconforming items is less than or equal to the acceptance number, the entire lot is to be accepted. Only those items in the sample that were found nonconforming are not accepted. If, on the other hand, the number of nonconforming items is equal to or greater than the rejection number, the entire lot is not accepted. For reduced inspection, the rejection number may be more than one unit greater than the acceptance number. In this case, a number of nonconforming items may be found that is greater than the acceptance number and smaller than the rejection number. In this situation, the lot is to be accepted and on subsequent lots normal inspection is reinstated.

EXAMPLE 9

Screws are to be inspected to determine whether they have a slot. If the slot is missing, the screw is nonconforming. Suppose that a single sampling plan of ISO 2859-1 is to be used. The agreement specifies an AQL of 0,65 % and the use of normal inspection, level II. The lot size is 3 000. This calls for code letter K. Table II-A of ISO 2859-1:1989 indicates:

| | |
|---|---|
| sample size | n = 125 items |
| acceptance number | Ac = 2 nonconforming items |
| rejection number | Re = 3 nonconforming items |

A sample of 125 screws is drawn at random from the lot and inspected. One screw is found that has no slot, but since 1 is less than the acceptance number (Ac = 2), the lot is accepted but the nonconforming screw is removed.

Single sampling plans such as that in example 9 are simple to operate. For sampling inspection to perform satisfactorily, the sample has to be selected in a random manner. This implies that each item in the lot has an equal probability of being chosen as a part of the sample. When items are large, packaged in individual packages, and when there are containers holding large quantities, it is difficult to accomplish this random selection. In all cases, it is important that the method of sample selection be specified rather than left to the inspector.

## 2.19 Operating characteristic (OC) curves

Each sampling plan has an operating characteristic curve which clearly demonstrates its properties. For the single sampling plan:

| | |
|---|---|
| sample size | n = 200 items |
| acceptance number | Ac = 7 nonconforming items |
| rejection number | Re = 8 nonconforming items |

The operating characteristic curve is given in Figure 2. The horizontal scale indicates the quality level of the production process. The vertical scale indicates the corresponding percent of lots which, on average, will be accepted from this process if this sampling plan is applied.

In practice, the quality level of a submitted lot is not known. If it were, lots could be sentenced directly without inspection. The operating characteristic curve shows what the sampling plan will do under particular circumstances. More precisely, the operating characteristic curve shows the probability of acceptance for lots with assumed values of the quality level, i.e. the percent nonconforming.

If, in the example in Figure 2, lots with no nonconforming items are inspected, the acceptance rate will be 100 %; i.e. no nonconforming item can be found and the acceptance number of 7 cannot be exceeded. If the quality level is 2,3 % nonconforming items, the operating characteristic curve of Figure 2 shows that the acceptance rate will be 90 %; i.e. on average 9 out of 10 lots will be accepted, and 1 out of 10 lots will not be accepted.

ISO 2859-1 presents operating characteristic curves of sampling inspection plans for percent nonconforming items and for nonconformities per 100 items. These OC curves show the average percentage of lots accepted as an ordinate plotted against the percentage of nonconforming items or the number of nonconformities per 100 items in the process quality as the abscissa. For percent nonconforming, they have been calculated based on the binomial distribution when the single sample size is equal to or less than 80. For sample sizes greater than 80, the binomial distribution is replaced by the Poisson approximation to the binomial distribution. For nonconformities per 100 items, the Poisson distribution is appropriate and has been used when calculating the OC curves for these plans.

The Poisson distribution is based on the assumption that nonconformities occur independently with constant expectation. This assumption holds in many cases. Any substantial departure from this assumption yields distributions with greater variance than that of the Poisson distribution. In these cases, the consumer's protection is somewhat better than that indicated by the operating characteristic curves.

The double and multiple sampling plans of ISO 2859-1 (see **2.20** and **2.21**) have been chosen to have OC curves that approximately match those of the single sampling plans corresponding to the same AQL and sample size code letter.

An understanding of the implications of the operating characteristic curve is essential for such tasks as setting the inspection level and putting limits on lot sizes. It is the comparison of operating characteristic curves that enables one to compare one single sampling plan with another.

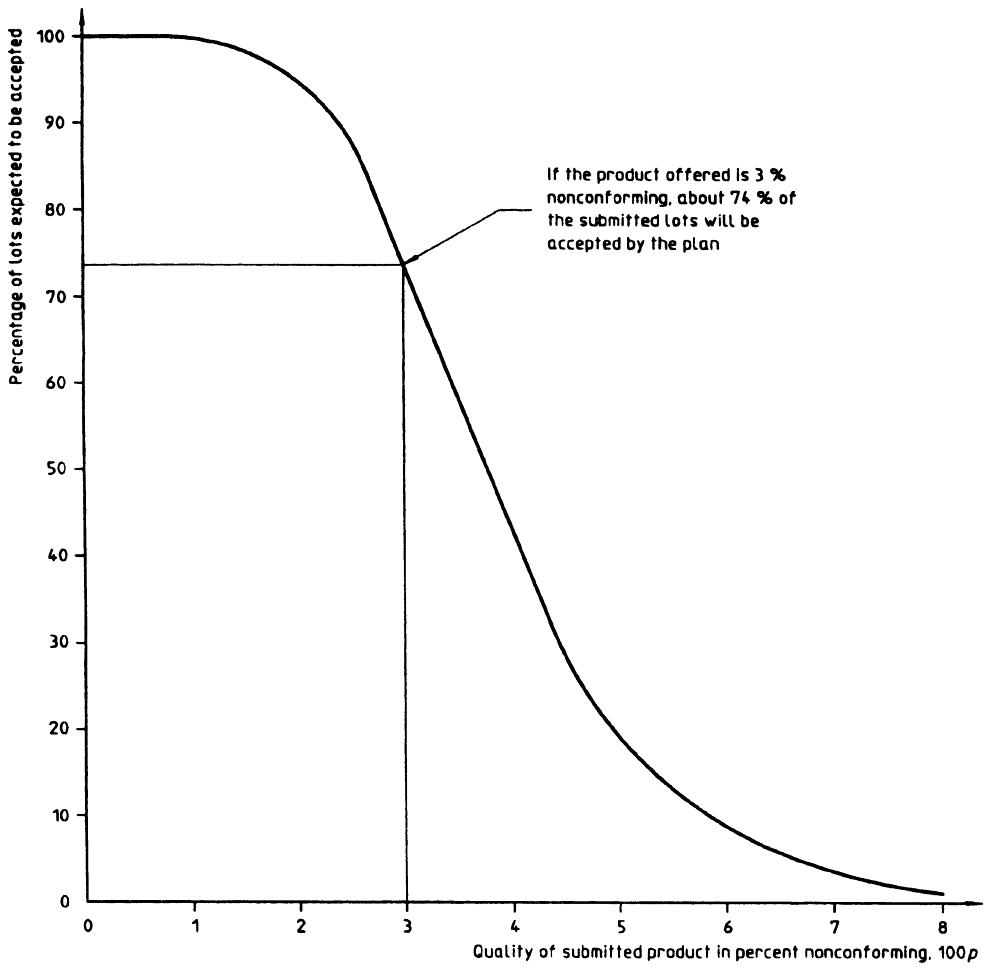
Note that both scales in Figure 2 concern long-term properties of the sampling plan. The horizontal scale gives the process average, not the quality of a single lot, and the vertical scale gives the percentage of lots expected to be accepted, not the percentage that will be accepted in any particular series of lots. An operating characteristic curve drawn in this way is thus appropriate to the lot-by-lot sampling schemes of ISO 2859-1, or the skip-lot schemes of ISO 2859-3. If one is concerned with an isolated lot, such an operating characteristic will show, approximately, how the probability that the lot will be accepted depends on the quality of that lot.

ISO 2859-2 gives operating characteristics which will be of interest to producers who are concerned with their chances of success in the long term, and it also gives exact values of the probability of acceptance for the isolated lot plans.

Comment on OC curves and the use of Table X of ISO 2859-1:1989 is given in **3.19**.

## 2.20 Double sampling

Double sampling is a procedure in which a first sample is taken that is smaller than would be taken for single sampling. If the quality of the first sample is sufficiently good, the lot will be accepted or, if sufficiently bad, the lot will fail to be accepted. Only in the case of intermediate quality is a second sample taken and examined in order to decide whether to accept or not accept the lot.




Sample size: $n = 200$
Acceptance number: Ac = 7
Rejection number: Re = 8

**Figure 2 — Operating characteristic curve of a single sampling plan with sample size 200 and acceptance number 7** (ISO 2859.1)

An example of matching single and double sampling plans is given below.

EXAMPLE 10

A product is to meet an AQL of 0,65 %. Lot acceptance is to be based on general inspection level II. Normal inspection is to be used. The lot size is 5 000 items.

Table I of ISO 2859-1:1989 indicates that the sample size code letter is L.

A single sampling plan according to Table II-A of ISO 2859-1:1989 requires a sample of 200 items with

| | |
|---|---|
| acceptance number | Ac = 3 nonconforming items |
| rejection number | Re = 4 nonconforming items |

The equivalent double sampling plan obtained from Table III-A is as follows:

| | |
|---|---|
| first sample size | n1 = 125 items |
| acceptance number | Ac = 1 nonconforming item |
| rejection number | Re = 4 nonconforming items |
| | |
| second sample size | n2 = 125 items |
| combined sample size | n = 250 items |
| acceptance number | Ac = 4 nonconforming items |
| rejection number | Re = 5 nonconforming items |

This means if 0 or 1 nonconforming items are found in a first sample of 125, the lot is to be accepted without a second sample being inspected; if 4 or more nonconforming items are found, the lot is sentenced as not acceptable without inspecting a second sample, if, however, the first sample of 125 contains 2 or 3 nonconforming items, a second sample of 125 has to be taken and the decision then depends upon the total number of nonconforming items in both samples combined: acceptance for 4 nonconforming items or less; non-acceptance for 5 or more.

In this example and in all the double sampling plans of the tables in ISO 2859-1, the first and second sample sizes are equal (see **3.16**).

## 2.21 Multiple sampling

ISO 2859-1 multiple sampling plans use up to seven samples. The decision to accept or not accept is usually made well before reaching the seventh sample.

Using the same lot size as is used in **2.20**, the sampling plan for code letter L and an AQL of 0,65 % is found in Table IV-A of ISO 2859-1:1989 and is as given in Table 1.

The rules for multiple sampling are an obvious extension of those for double sampling so they need not be specified here in detail. The only new feature is that sometimes the symbol # is to be found in place of an acceptance number. This indicates that acceptance is not permitted, so only two decisions are possible at this stage: that of non-acceptance or continuing with the examination of a further sample.

All multiple sampling plans in ISO 2859-1 have all the seven sample sizes equal, as in this illustration (see **3.16**).

## 2.22 Sequential sampling

In sequential sampling, items are randomly sampled from a lot and inspected one after another. A cumulative count is kept of the number of items inspected and the number of nonconforming items. The decision rules provide for acceptance or non-acceptance of the lot as soon as the evidence is sufficiently strong one way or the other. To avoid the possibility of sampling continuing indefinitely with no decision being reached, a curtailment rule is provided. Sampling is stopped at a specified sample size. Criteria are provided for making a decision at this stage.

ISO 8422 contains procedures for calculating sequential sampling plans by attributes. These plans can be chosen to have the same producer's and consumer's risk as the plans of ISO 2859-1.

The operation of a sequential sampling plan can be illustrated graphically by the following example.

**Table 1 — Multiple sampling plan for code letter L and an AQL of 0,65 %**

| Sample | Sample size | Cumulative sample size | Acceptance number | Rejection number |
|---|---|---|---|---|
| 1st | 50 | 50 | # | 3 |
| 2nd | 50 | 100 | 0 | 3 |
| 3rd | 50 | 150 | 1 | 4 |
| 4th | 50 | 200 | 2 | 5 |
| 5th | 50 | 250 | 3 | 6 |
| 6th | 50 | 300 | 4 | 6 |
| 7th | 50 | 350 | 6 | 7 |

NOTE The symbol # means that an accept decision is not permitted at this sample size.

EXAMPLE 11

Figure 3 shows the sequential sampling plan corresponding to single and double sampling plans described in **2.20** and to the multiple sampling plan described in **2.21**. In this example, the total number of items inspected is curtailed at 300. The decision to accept is then based on whether 5 or fewer nonconforming items have been found. If 6 or more nonconforming items have been found, the lot is not acceptable. In the example, nonconforming items occurred at positions 30 and 100. The indecision zone was breached for the first time when acceptance region was entered after the 198th item was inspected. The lot was sentenced "acceptable".

## 2.23 Skip-lot sampling

When a continuing series of lots is being received and the quality of product submitted to an inspection has regularly proven to be considerably better than the AQL, and provided certain criteria have been met, it is possible to initiate skip-lot inspection.

The skip-lot procedure described in ISO 2859-3 systematically qualifies product for skip-lot inspection. When product qualifies to specific criteria, a fraction (*viz.* 1/2, 1/3, 1/4 or 1/5) of the lots is sampled. The smaller fractions are only allowable when the product exhibits a quality level markedly better than the AQL. A sampling frequency of 1/5 is not available initially. Each lot is chosen at random to be sampled or accepted without sampling. If the quality, as measured at sampling inspection, deteriorates, a return is made to sampling each lot until the product re-qualifies.

The advantages of random sampling and calculable risks are maintained.

The amount of inspection and the resulting cost of skip-lot sampling are sometimes less than for reduced sampling as per ISO 2859-1.

True randomness of the process for sample selection is preserved.

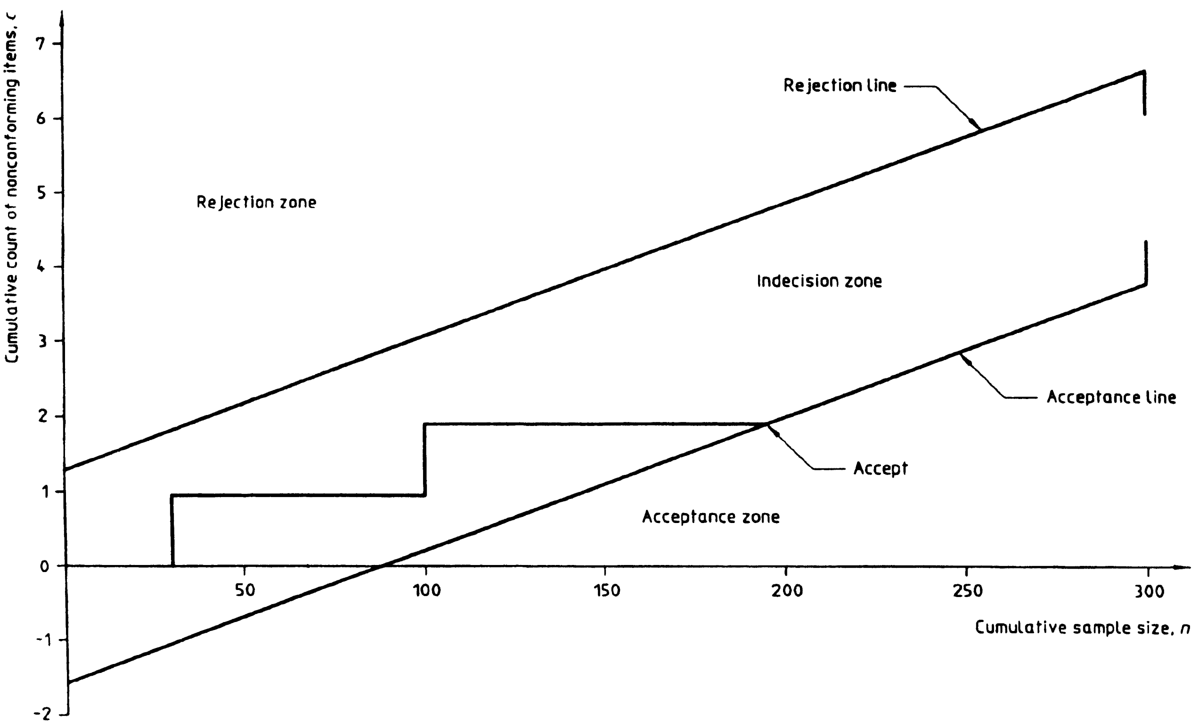



**Figure 3 — Operation of the curtailed sequential sampling plan of ISO 8422 corresponding to an AQL of 0,65 %, sample size code letter L**

## 2.24 Comparison of single, double, multiple and sequential sampling

### 2.24.1 Equivalent plans

If the acceptance number in the single sampling plan is greater than zero, then it is possible to find a double, multiple or sequential sampling plan with an operating characteristic curve close to that of the single sampling plan. Hence, except for those single plans with acceptance number zero, there is no reason to choose between single, double, multiple or sequential sampling on the basis of the operating characteristic curve. Neither is there reason to prefer one to another for all possible situations. The balance of advantages and disadvantages sometimes favours one, sometimes another of the sampling procedures. The characteristics that should be taken into account are as follows.

**a) Simplicity**

Single sampling is the easiest to describe and administer. Double sampling requires more administration to arrange for the second sample to be made available when required. Multiple and sequential sampling are obviously even more complicated. Sometimes the attraction of simplicity is the major consideration in the selection of the sampling plan. There will be other occasions when the psychological attraction of being able to take a second sample in apparently marginal cases will favour double sampling plans.

**b) Variability in the amount of sampling inspection**

In single sampling, the sample size is fixed and the amount of inspection effort required to reach a decision is known in advance. For the other types of sampling, the number of items tested varies according to the results from the early samples. It is possible to calculate an average amount of sampling inspection and the average cost of inspection for any given input quality. This varies with the quality, being least for both very good and for very poor quality. In addition to the uncertainty associated with the unknown input quality, but also even when the input quality is known, there is the uncertainty due to the variation of the amount of sampling inspection about this average. This uncertainty can lead to problems in arranging for sufficient resources to be made available for the inspection required. If insufficient resources are available, the result is delayed. In the contrary case there will be inefficient use of resources. In some situations, the variable inspection load will often be considered a small price to pay for the significant reduction in the average total inspection cost.

**c) Ease of drawing sample items**

Sometimes it is easy to draw a second sample and to draw two samples is no more trouble than to draw one sample of the combined size. At other times, however, the situation arises where the drawing of sample items forms a large part of the inspection task and here, having disturbed the lot to draw one sample, it is hardly feasible to disturb it again to draw another sample. In these cases, single sampling is usually the best plan. There is, of course, the alternative possibility of drawing a sample of the maximum size that could be needed and then inspecting according to the preselected double, multiple or sequential plan. This may give little cost-saving compared with the single plan due to problems in returning uninspected items to the lot.

**d) Duration of test**

If a test is of long duration and it is possible to apply it to a number of items simultaneously, it will usually be better to do so rather than to risk finding that at the end of the test of a first sample the result is inconclusive and that a second sample, or even more, is needed, therefore at least doubling the time taken. This is another case where single sampling is usually the best, provided that the whole of the single sample size can be tested at once. However, if only one or two articles can be tested at one time, multiple (or sequential) sampling may be preferable.

EXAMPLE 12

Tinned meat is to be tested for keeping qualities by storing a number of tins for 3 weeks under certain atmospheric conditions.

To achieve a desired OC curve, the choice might perhaps lie between a single sample of 80 tins, a double plan with samples each of 50 tins, and a seven-stage multiple plan with samples each of 20 tins. If single sampling is used, the answer will be available 3 weeks after the test is started; under double sampling, the result might be available in 3 weeks, but might require 6 weeks instead; under multiple sampling, nearly 5 months might be required in an unlucky event.

Single sampling will probably be chosen in these circumstances.

EXAMPLE 13

A destructive inspection is to be performed. All the articles in the lot are available at the testing station and the testing apparatus can take only one article at a time. As the principal cost of the test is the destruction of the article, it is desirable to destroy as few as possible consistent with the desired OC curve.

As the articles in the sample have to be tested one at a time, the use of sequential rather than single sampling will probably save time as well as cut down the average sample size and would be well worth considering.

**e) Multiple nonconformities**

The more complicated the product in terms of the number of possible nonconformities and number of classes of nonconformities, the more involved double or multiple sampling becomes. Efficient use of labour and inspection equipment is difficult if the first sample has to be inspected for all features, a second sample only for some features, and possibly a third sample only for some of those. In general, it can be said that a complicated inspection favours a simple sampling plan, whereas, where the inspection is simpler, a more complicated sampling plan may pay rich dividends.

The operating characteristic curve for the single sampling plan with sample size 200, acceptance number 3, and rejection number 4, and the equivalent double and multiple plans discussed in **2.20** and **2.21** are shown in Figure 4. The match is not exact, but is good enough for most practical purposes. The equivalent sequential plan is also matched to the OC curve for the single sampling plan but is not shown in order to avoid overcrowding. The operating characteristic curves of the sequential and single sampling plans are virtually indistinguishable from each other.

**2.24.2 Average sample size**

Subclauses **2.18** to **2.22** have described single, double, multiple and sequential sampling. For comparative purposes, it is helpful to consider the average sample size that would be needed in a long run of sampling at different average product quality. This produces an average sample size curve which is indicative of the relative efficiency of the several sampling systems. These curves indicate the number of items to be examined on average before arriving at a decision to accept or reject. Figure 5 shows the average sample sizes for the set of equivalent single, double, multiple and sequential sampling plans given in Table 2 and featured in Figure 4.

On average, the number of items to be examined before reaching a decision is largest when single sampling is used. The greatest reduction in sample size when using double, multiple or sequential sampling occurs when lots are of very good quality or very bad quality.

For good or bad quality, the average saving in inspection can be substantial, but the actual number of items to be inspected for a particular lot when using a double, multiple or sequential sampling plan may exceed that for the corresponding single plan. This is most likely to occur when quality is at an intermediate value, e.g. two or three times the AQL.

It is for these reasons that single sampling may be preferred in some instances, for example, when the test duration is long and all items can be tested at the same time. On the other hand, when the tests can only be done one at a time or are destructive, double, multiple or sequential sampling can offer a substantial advantage (see examples 10 and 11).

For double and multiple plans, there is an upper limit to the number of items to be inspected. For sequential plans, there is generally no such limit unless the truncation rule has been invoked to restrict the potential number of items inspected. ISO 8422 and ISO 8423 provide for curtailment of sample size.

Double, multiple and sequential sampling offer the opportunity for significant savings in sample size, but they require more administrative control. When apparatus for semi-automatic use is available, automated sequential sampling offers an opportunity for increased efficiency and economy, particularly when destructive tests are performed.

Average sample size curves for double and multiple plans are given in ISO 2859-1. For sequential sampling plans by attributes, the average sample sizes are tabulated in ISO 8422. For sequential sampling plans by variables for known process standard deviation corresponding to the single sampling plans of ISO 3591, the average sample sizes are given in ISO 8423. ISO 3951 does not contain double, multiple or sequential sampling plans.

## 2.25 Drawing of samples

In acceptance sampling, a lot is sentenced on the quality of the sample. Hence the sample needs to be representative of the lot. What is demanded is a random sample and not a biased one.

The intuitive attempt to draw a random sample often gives biased results.

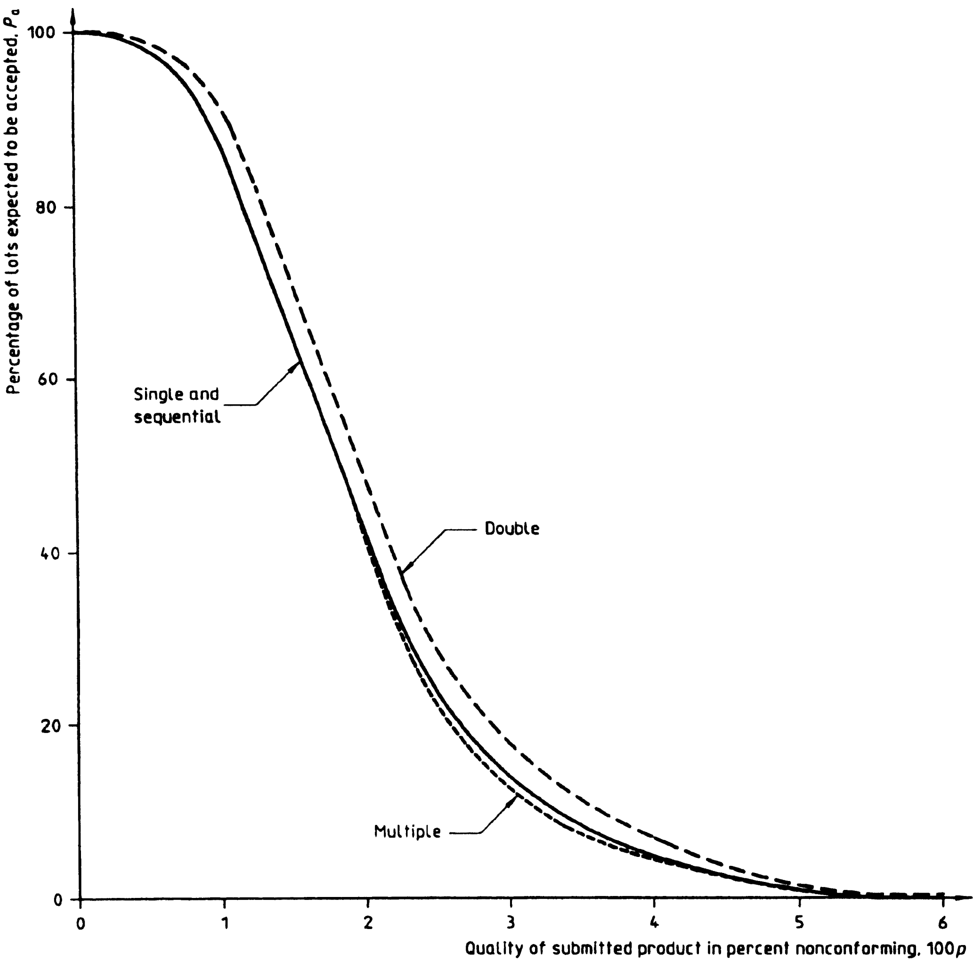



**Figure 4 — Comparison of operating characteristic curves for single, double, multiple and sequential sampling plans (code letter L, AQL = 0,65%)**

**Table 2 — Equivalent sampling plans for code letter L and an AQL of 0,65 %**

| Type of sampling plan | Sample size(s) | | Ac | Re |
|---|---|---|---|---|
| Single | Sample | $n$ = 200 | 3 | 4 |
| Double | First sample | $n$ = 125 | 1 | 4 |
| | Combined 1st and 2nd sample | $n$ = 250 | 4 | 5 |
| Multiple | First sample | $n$ = 50 | # | 3 |
| | Cumulative sample size | $n$ = 100 | 0 | 3 |
| | | $n$ = 150 | 1 | 1 |
| | | $n$ = 200 | 2 | 5 |
| | | $n$ = 250 | 3 | 6 |
| 6 | | $n$ = 300 | 4 | 6 |
| | | $n$ = 350 | 6 | 7 |
| Sequential | See **2.22** | | | |
| NOTE The symbol # means that acceptance is not permitted at this sample size. | | | | |

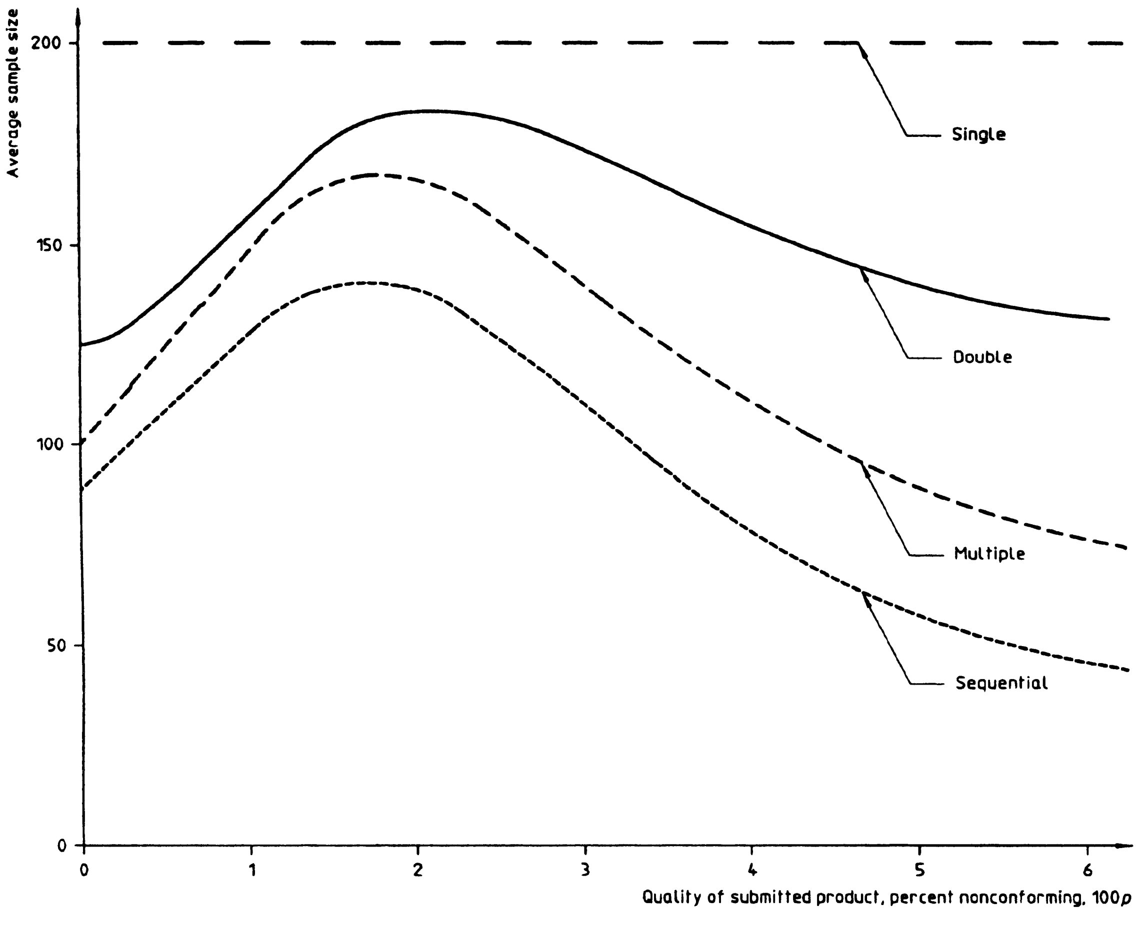



**Figure 5 — Comparison of average sample sizes for single, double, multiple and sequential sampling** (for acceptance plans of Figure 4)

For example, people drawing items, supposedly at random, from a box will usually draw too many from the middle and the corners will not be adequately represented. When it is pointed out that they are taking too few from the corners, they will then often start to take too many from the corners. The simple randomness of giving every combination and equal chance is very elusive, and the extra trouble of using random numbers where possible is worthwhile.

Provided that the items can be ordered or numbered, the selection of a random sample can be made using Table 3 or any approved table of random numbers.

EXAMPLE 14

A sample of size 8 is to be drawn from a lot of 5 000. The articles in the lot are labelled with numbers from 1 to 5 000 and, starting at the top of the first column of Table 3, the articles to be drawn for the sample are numbers 110, 4 148, 2 403, 1 828, 2 267, 2 985, 4 313 and 4 691 (the numbers 5 327, 5 373, 9 244 etc. being ignored as corresponding articles would not be found in the lot).

**Table 3 — Random sampling numbers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0110 | 9140 | 2804 | 8046 | 7142 | 6277 | 6210 | 8627 | 3209 | 6845 |
| 5327 | 3946 | 6289 | 6117 | 0060 | 2827 | 6546 | 2738 | 8760 | 6604 |
| 5373 | 8259 | 4956 | 8185 | 0135 | 8640 | 7410 | 6335 | 0831 | 2774 |
| 9244 | 9452 | 8324 | 8062 | 9817 | 9853 | 7479 | 9559 | 4264 | 6919 |
| 4148 | 3948 | 5399 | 8687 | 3568 | 4046 | 4558 | 0705 | 5075 | 4440 |
| 2403 | 4351 | 8240 | 3554 | 3568 | 4701 | 7494 | 6036 | 7735 | 4082 |
| 1828 | 1956 | 1646 | 1370 | 9096 | 0738 | 8015 | 0513 | 6969 | 0949 |
| 7249 | 9634 | 4263 | 4345 | 0567 | 1272 | 5302 | 3352 | 7389 | 9976 |
| 7116 | 9731 | 2195 | 3265 | 9542 | 2808 | 1720 | 4832 | 2553 | 7425 |
| 6659 | 8200 | 4135 | 6116 | 3019 | 6223 | 7323 | 0965 | 8105 | 4394 |
| 2267 | 0362 | 5242 | 0261 | 7990 | 8886 | 0375 | 7577 | 8422 | 5230 |
| 9460 | 9813 | 8325 | 6031 | 1102 | 2825 | 4899 | 1599 | 1199 | 0909 |
| 2985 | 3541 | 6445 | 7981 | 8796 | 9480 | 2409 | 9456 | 7725 | 0183 |
| 4313 | 0666 | 2179 | 1031 | 7804 | 8075 | 8187 | 6575 | 0065 | 2170 |
| 6930 | 5368 | 4520 | 7727 | 2536 | 4166 | 7653 | 0448 | 2560 | 4795 |
| 8910 | 3585 | 5655 | 1904 | 0681 | 6310 | 0568 | 3718 | 3537 | 8858 |
| 8439 | 1052 | 5883 | 9283 | 1053 | 5667 | 0572 | 0611 | 0100 | 5190 |
| 4691 | 6787 | 4107 | 5073 | 8503 | 6875 | 7525 | 8894 | 7426 | 0212 |
| 1034 | 1157 | 5888 | 0213 | 2430 | 7397 | 7204 | 6893 | 7017 | 7038 |
| 7472 | 4581 | 3837 | 8961 | 7931 | 6351 | 1727 | 9793 | 2142 | 0816 |
| 2950 | 7419 | 6874 | 1128 | 5108 | 7643 | 7335 | 5303 | 2703 | 8793 |
| 1312 | 7297 | 3848 | 4767 | 5386 | 7361 | 2079 | 3197 | 8904 | 4332 |
| 8734 | 4921 | 6201 | 5057 | 9228 | 9938 | 5104 | 6662 | 1617 | 2323 |
| 2907 | 0737 | 8496 | 7509 | 9304 | 7112 | 5528 | 2390 | 7736 | 0475 |
| 1294 | 4883 | 2536 | 2351 | 5860 | 0344 | 2595 | 4880 | 5167 | 5370 |

**Table 3 — Random sampling numbers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0430 | 5819 | 7017 | 4512 | 8081 | 9198 | 9786 | 7388 | 0704 | 0138 |
| 5632 | 0752 | 8287 | 8178 | 8552 | 2264 | 0658 | 2336 | 4912 | 4268 |
| 7960 | 0067 | 7837 | 9890 | 4490 | 1619 | 6766 | 6148 | 0370 | 8322 |
| 5138 | 6660 | 7759 | 9633 | 0924 | 1094 | 5103 | 1371 | 2874 | 5400 |
| 8615 | 7292 | 1010 | 9987 | 2993 | 5116 | 7876 | 7215 | 9715 | 3906 |
| 4968 | 8420 | 5016 | 1391 | 8711 | 4118 | 3881 | 9840 | 5843 | 0751 |
| 9228 | 3232 | 5804 | 8004 | 0773 | 7886 | 0146 | 2400 | 6957 | 8968 |
| 9657 | 9617 | 1033 | 0469 | 3564 | 3799 | 2784 | 3815 | 3611 | 8362 |
| 9270 | 5743 | 8129 | 8655 | 4769 | 2900 | 6421 | 2788 | 4858 | 5335 |
| 8206 | 3008 | 7396 | 0240 | 0524 | 3384 | 6518 | 4268 | 5988 | 9096 |
| 1562 | 7953 | 0607 | 6254 | 0132 | 3860 | 6630 | 2865 | 9750 | 9397 |
| 1568 | 4342 | 5173 | 3322 | 0026 | 7513 | 1743 | 1299 | 1340 | 6470 |
| 5697 | 9273 | 8609 | 8442 | 1780 | 1961 | 7221 | 5630 | 8036 | 4029 |
| 3186 | 0656 | 3248 | 0341 | 9308 | 9853 | 5129 | 3956 | 4717 | 7594 |
| 3275 | 7697 | 1415 | 5573 | 9661 | 0016 | 4090 | 2384 | 7698 | 4588 |
| 7931 | 1949 | 1739 | 3437 | 6157 | 2128 | 6026 | 2268 | 5247 | 2987 |
| 5956 | 2912 | 2698 | 5721 | 1703 | 2321 | 8880 | 3268 | 7420 | 2121 |
| 1866 | 7901 | 4279 | 4715 | 9741 | 2674 | 7148 | 8392 | 2497 | 8018 |
| 2673 | 7071 | 4948 | 8100 | 7842 | 8208 | 3256 | 3217 | 8331 | 7256 |
| 7824 | 5427 | 0957 | 6076 | 2914 | 0336 | 3466 | 0631 | 5249 | 7289 |
| 2251 | 0864 | 0373 | 7808 | 1256 | 1144 | 4152 | 8262 | 4998 | 3315 |
| 7661 | 8813 | 5810 | 2612 | 3237 | 2829 | 3133 | 4833 | 7826 | 1897 |
| 6651 | 6718 | 1088 | 2972 | 0673 | 8440 | 3154 | 6962 | 0199 | 2604 |
| 2917 | 4989 | 9207 | 4484 | 0916 | 9129 | 6517 | 0889 | 0137 | 9055 |
| 5970 | 3582 | 2346 | 8356 | 0780 | 4899 | 7204 | 1042 | 8795 | 2435 |
| 1564 | 8048 | 6359 | 8802 | 2860 | 3546 | 3117 | 7357 | 9945 | 5739 |
| 6022 | 9676 | 5768 | 3388 | 9918 | 8897 | 1119 | 9441 | 8934 | 8555 |
| 8418 | 9906 | 0019 | 0550 | 4223 | 5586 | 4842 | 8786 | 0855 | 5650 |
| 5948 | 1652 | 2545 | 3981 | 2102 | 3523 | 7419 | 2359 | 0381 | 8457 |
| 6945 | 3629 | 7351 | 3502 | 1760 | 0550 | 8874 | 4599 | 7809 | 9474 |

**Table 3 — Random sampling numbers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0370 | 1165 | 8035 | 4415 | 9812 | 4312 | 3524 | 1382 | 4732 | 2303 |
| 6702 | 6457 | 2270 | 8611 | 8479 | 1419 | 0835 | 1866 | 1307 | 4211 |
| 3740 | 4721 | 3002 | 8020 | 0182 | 4451 | 9389 | 1730 | 3394 | 7094 |
| 3833 | 3356 | 9025 | 5749 | 4780 | 6042 | 3829 | 8458 | 1339 | 6948 |
| 8683 | 7947 | 4719 | 9403 | 7863 | 0701 | 9245 | 5960 | 9257 | 2588 |
| 6794 | 1732 | 4809 | 9473 | 5893 | 1154 | 0067 | 0899 | 1184 | 8630 |
| 5054 | 1532 | 9498 | 7702 | 0544 | 0087 | 9602 | 6259 | 3807 | 7276 |
| 1733 | 6560 | 9758 | 8586 | 3263 | 2532 | 6668 | 2888 | 1404 | 3887 |
| 6609 | 6263 | 9160 | 0600 | 4304 | 2784 | 1089 | 7321 | 5618 | 6172 |
| 3970 | 7716 | 8807 | 6123 | 3748 | 1036 | 0516 | 0607 | 2710 | 3700 |
| 9504 | 2769 | 0534 | 0758 | 9824 | 9536 | 7825 | 2985 | 3824 | 3449 |
| 0668 | 9636 | 6001 | 9372 | 8746 | 1579 | 6102 | 7990 | 4526 | 3429 |
| 4364 | 0606 | 4355 | 2395 | 2070 | 8915 | 8461 | 9820 | 6811 | 5873 |
| 8875 | 3041 | 7183 | 2261 | 7210 | 6072 | 7128 | 0825 | 8281 | 6815 |
| 4521 | 3391 | 6695 | 5986 | 2416 | 7979 | 8106 | 7759 | 6379 | 2101 |
| 5066 | 1454 | 9642 | 8675 | 8767 | 0582 | 0410 | 5515 | 2697 | 1575 |
| 9138 | 5003 | 8633 | 2670 | 7575 | 4021 | 0391 | 0118 | 9493 | 2291 |
| 0975 | 1836 | 7629 | 5136 | 7824 | 3916 | 0542 | 2614 | 6567 | 3015 |
| 1049 | 9925 | 3408 | 3029 | 7244 | 1766 | 1013 | 0221 | 8492 | 3801 |
| 0682 | 1343 | 7454 | 9600 | 8598 | 9953 | 5773 | 6482 | 4439 | 6708 |
| 0263 | 4909 | 9832 | 0627 | 1155 | 4007 | 0446 | 6988 | 4699 | 1740 |
| 2733 | 3398 | 7630 | 3824 | 0734 | 7736 | 8465 | 0849 | 0459 | 8733 |
| 1441 | 2684 | 1116 | 0758 | 5411 | 3365 | 4489 | 6241 | 6413 | 3615 |
| 5014 | 5616 | 1721 | 8772 | 4605 | 0388 | 1399 | 5993 | 7459 | 4445 |
| 3745 | 5956 | 5512 | 8577 | 4178 | 0031 | 3090 | 2296 | 0124 | 5896 |
| 8384 | 8727 | 5567 | 5881 | 3721 | 1898 | 3758 | 7236 | 6860 | 1740 |
| 9944 | 8361 | 7050 | 8783 | 3815 | 9768 | 3247 | 1706 | 9355 | 3510 |
| 3045 | 2466 | 6640 | 6804 | 1704 | 8665 | 2539 | 2320 | 9831 | 9442 |
| 5939 | 5741 | 7210 | 0872 | 3279 | 3177 | 6021 | 2045 | 0163 | 3706 |
| 4294 | 1777 | 5386 | 7182 | 7238 | 8408 | 7674 | 1719 | 9068 | 9921 |

**Table 3 — Random sampling numbers**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3787 | 2516 | 2661 | 6711 | 9240 | 5994 | 3068 | 5524 | 0932 | 5520 |
| 4764 | 2339 | 4541 | 5415 | 6314 | 7979 | 3634 | 5320 | 5400 | 6714 |
| 0292 | 9574 | 0285 | 4230 | 2283 | 5232 | 8830 | 5662 | 6404 | 2514 |
| 7876 | 1662 | 2627 | 0940 | 7836 | 3741 | 3217 | 8824 | 7393 | 7306 |
| 3490 | 3071 | 2967 | 4922 | 3658 | 4333 | 6452 | 9149 | 4420 | 6091 |
| 3670 | 8960 | 6477 | 3671 | 9318 | 1317 | 6355 | 4982 | 6815 | 0814 |
| 3665 | 2367 | 8144 | 9663 | 0990 | 6155 | 4520 | 0294 | 7504 | 0223 |
| 3792 | 0557 | 8489 | 8446 | 8082 | 1122 | 1181 | 8142 | 7119 | 3200 |
| 2618 | 2204 | 9433 | 2527 | 5744 | 9330 | 0721 | 8866 | 3695 | 1081 |
| 8972 | 8829 | 0962 | 5597 | 9834 | 5857 | 9800 | 7375 | 9209 | 0630 |
| 7305 | 8852 | 1688 | 3571 | 3393 | 2990 | 9488 | 8883 | 2476 | 9136 |
| 1794 | 4551 | 1262 | 4845 | 4039 | 7760 | 1565 | 4745 | 1178 | 8370 |
| 3179 | 1304 | 7767 | 4769 | 7373 | 5195 | 5013 | 6894 | 5734 | 5852 |
| 2930 | 3828 | 7172 | 3188 | 7487 | 2191 | 1225 | 7770 | 3999 | 0006 |
| 8418 | 9627 | 7948 | 6243 | 1176 | 9393 | 2252 | 0377 | 9798 | 8648 |

The following should be noted with regard to the use of a table of random sampling numbers.

a) It is incorrect to always start at the top of the first column. For each sample to be drawn, the best procedure is to start from an arbitrary point and work through the table either up or down columns or across rows.

b) There is no need to read the numbers as having four figures. If the lot size were 1 000 or less, the first three figures would be adequate, and would be read as 11, 532, 537, etc. Sometimes two figures are enough, sometimes more than four are required. As many or as few as desired may be combined.

It has to be recognized, however, that the use of random numbers is not always easy. If the lot consists of a large box of small articles, it may be quite impracticable to give each one a number. In such circumstances, intuitive random sampling is probably all that can be done, but if intuition is modified by knowledge of what would be done if it were possible, this will help to obtain better results. Knowing that every possible combination has to have an equal chance makes it clear at once that the articles have to be taken out of the box to make them all equally available before the sample is drawn, and also that any apparent quality of the articles should be ignored. There should not be deliberate choosing of articles which appear good or bad.

There is an alternative to simple random sampling which is allowable, indeed desirable, where appropriate, and it may be used whether or not random numbers are used. This alternative is known as representative or stratified sampling. This is appropriate whenever a lot can be split into sub-lots according to some logical criterion. Note that the criterion has to be a logical one; splitting into sub-lots at random will not help. The sample is drawn by taking a sub-sample from each sub-lot proportionate in size to the size of the sub-lot. The sub-samples have to be drawn at random from within the sub-lots (using random numbers, if possible) and finally the sub-samples are combined to make up the complete sample before inspection.

EXAMPLE 15

A sample of 125 is to be drawn from a lot which has been delivered in two boxes, half the lot being in each box. It is decided to make each box a sub-lot. A sample of 62 is drawn from one box and of 63 from the other, these two samples being combined to form the required sample of 125. (The box to supply the one extra item should preferably be chosen at random.)

If, instead of each box containing half the lot, one box had contained two-thirds and the other one-third, then 83 would have been drawn from the first box and 42 from the second, as being the nearest whole numbers to two-thirds and one-third of 125.

When using double or multiple sampling, it is occasionally convenient to draw the sample at random and inspect it, then draw the second sample if required, and so on. In this case, the random sampling techniques are as described above, and no extra difficulty arises. Sometimes it is more convenient to draw, the maximum sample that might be required and divide it into first sample, second sample, etc. before inspection. In this case, it is most important that, in addition to drawing a sample at random from the lot to make up the maximum sample, the first, second, etc. samples be drawn at random from the maximum sample. It is particularly important to remember this point when stratified sampling is used. It would be quite wrong, for instance, to allow all the first sample to come from one sub-lot and all of the second sample from another sub-lot.

Truly random sampling provides an equal opportunity for each item in the lot to be included in the sample and this is implicit in the use of all plans.

## 2.26 Sampling plans, schemes and systems

A sampling plan is a set of rules by which a lot is to be inspected and sentenced.

A sampling scheme is a combination of sampling plans with switching procedures.

A sampling system is a collection of sampling schemes, each with its own rules for changing plans, together with criteria by which appropriate schemes may be chosen.

For example parts 1, 2 and 3 of ISO 2859 and ISO 8422 provide sampling systems. They contain many sampling schemes and describe the conditions (AQL, lot size, inspection level, etc.) under which these sampling schemes are appropriate.

An example of a single sampling plan is:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 5 nonconforming items |
| rejection number | Re = 6 nonconforming items |

An example of a single sampling scheme is the following combination of normal, tightened and reduced inspection, with switching rules which are given in ISO 2859-1:

| | **Normal** | **Tightened** | **Reduced** |
|---|---|---|---|
| sample size | $n$ = 125 | $n$ = 125 | $n$ = 50 |
| acceptance number | Ac = 5 | Ac = 3 | Ac = 2 |
| rejection number | Re = 6 | Re = 4 | Re = 5 |

## 2.27 Distributional characteristics (binomial, Poisson and hypergeometric)

The operating characteristic curves for lots in a continuing series where the interest is in percent nonconforming are calculated by using the binomial distribution. The process is presumed to be operating under conditions where the average quality is 100$p$ %. The process is operating randomly while maintaining this average quality. Lots will then contain an average percent nonconforming of 100$p$, distributed in accordance with the binomial distribution.

Where a continuing series of lots is being sampled for nonconformities per 100 items, there is often no natural upper limit to the number of such nonconformities. In the instance where nonconformities per 100 items is used as the basis for acceptance, the Poisson distribution is the valid representation and is used as the basis for calculating the probability of acceptance of each lot in a series of lots.

The foregoing discussion applies to parts 1 and 3 of ISO 2859 and to ISO 8422.

When an isolated lot is sampled, the proper method for calculating the probability of acceptance is the hypergeometric distribution.

When the sample is a small proportion of the lot, the binomial distribution is a good approximation of the hypergeometric distribution. When the percent nonconforming is small and the sample size is large, the Poisson distribution is a good approximation to the binomial distribution.

NOTE 4 For a thorough treatment of this topic, texts on statistical methods and sampling should be referred to.

In developing the early issues of sampling tables, the hypergeometric distribution was not used due to the lengthy calculations required. This is no longer a problem so the hypergeometric distribution is used where appropriate.

# Section 3. The ISO 2859-1 system

## 3.1 Description of ISO 2859-1

ISO 2859-1 has an introductory text giving instructions for the use of the tables that it contains, but that text is kept as short as possible and gives only the essential minimum of guidance. The aim of this part of ISO 2859 is to expand these instructions and, by giving a detailed commentary and a number of examples, to clarify the methods of sampling inspection that make up the ISO 2859 sampling system.

The bold numbers in square brackets in this section are references to the relevant clause numbers in ISO 2859-1:1989.

ISO 2859-1 is designed for lot-by-lot inspection by attributes [**3.13**]. The system is particularly relevant for "external" inspection of a sequence of lots. Internal inspection, or an occasional isolated lot, may be covered by considering ISO 2859-2 which treats the standard as a collection of sampling plans rather than as a collection of sampling schemes.

The tables in ISO 2859-1 are designated by Roman numerals, with sub-divisions denoted by capital letters, for example Table I, Table II-A, Table II-B, etc. The tables in this part of ISO 2859 are designated by Arabic numerals, for example Table 1, Table 2, Table 3, etc. In this part of ISO 2859, tables will be referred to without specifying each time which document is the appropriate one, as this will be clear from the table numbers themselves.

The main purpose of the ISO 2859-1 system is to control the acceptance of products at a quality level which is equal to or better than the acceptable quality level (AQL) [**3.6**]. However, the designation of an AQL does not imply that the producer has the right knowingly to supply any nonconforming product.

One way, therefore, of regarding the AQL is that it is an index of the calculated risks that the consumer is prepared to accept in order to obtain the economic benefit of sampling inspection. If, however, the sampling risk cannot be accepted, or a suitable plan is not available, then the product has to be inspected 100 %. When sampling inspection is being used and the product is produced at a quality that is worse than the AQL, a sampling plan that has been well chosen should, by leading to the acceptance of an insufficient fraction of the lots, make it worthwhile for an improvement in quality to be made with the minimum delay. It follows that when production is under control at a suitable level, a quality better than the AQL can be expected.

It needs to be also understood that it is often difficult and expensive to be sure that a machine, a process or a production line is producing no nonconforming items. In practice, some percentage of nonconforming items is usually acceptable but this does not necessarily mean that all of them will find their way into the final product; some will be detected at later inspection stages and others may fail to assemble or function on test. The acceptable limit for percent nonconforming is often governed by economic considerations, as the consumer may be faced with the choice between a reasonably good article that he can afford or a better one that is beyond his means. It is frequently found that better control of the process can produce a much smaller percentage of nonconforming items. Under these conditions, better quality can cost less.

ISO 2859-1 may be considered as consisting of four parts, namely the text, the master tables (Tables I to IV), supplementary tables (Tables V to IX), and the extended tables (Tables X-A to X-S).

The text defines the terms used and gives rules for the operation of sampling inspection.

The right-hand pages of the extended tables repeat information already given in the master tables. It proves useful in practice to have this information available in two different forms of lay-out; sometimes one lay-out is the more useful, sometimes the other.

The scheme is based upon the use of the AQL concept, and the plans are indexed by AQL and by sample size. The sample size itself, however, is not used directly as an index, but is coded in the form of a "sample size code letter" (see **3.18**, where the reason for this is explained).

Double sampling and multiple sampling plans are given [**11.1.2** and **11.1.3**] which are equivalent to the single sampling plans in the sense of having matching OC curves.

Tables are given for normal inspection, tightened inspection and reduced inspection [clause **9**], together with rules for switching from one of these to another [**9.3**].

## 3.2 Preparing a specification for use in conjunction with ISO 2859-1

The particular specification for a product or service should be written in suitable form if the product is to be readily subjected to the ISO 2859-1 method of sampling inspection. The requirements in such a specification may be summarized as follows.

a) Each of the inspection and/or test requirements relating to the product has to be expressed in attribute form; if the feature inspected is measurable, decide whether to use a variables approach instead.

b) Each such requirement should have the following factors positively indicated:

1) the item of product;

2) a classification of characteristics, where applicable;

3) whether each nonconformity is to be separately considered for AQL or whether (and how) they are to be grouped;

4) the required AQL for each characteristic or group of characteristics;

5) the required inspection level for each nonconformity or group of nonconformities;

6) whether normal or tightened inspection is to be applied initially;

7) any limitations on lot size;

8) whether reduced inspection may be permitted;

9) what should be done if inspection is discontinued;

10) the designation of responsible authority.

In addition, the type of sampling plan (single, double, etc.) may be specified, if desired, but need not be. If production will be in isolated batches, then it may be preferable to specify the LQ value, instead of the AQL, for use with ISO 2859-2.

## 3.3 Classification of nonconformities and nonconforming items

For the case where acceptance sampling involves the evaluation of more than one quality characteristic, ISO 2859-1 gives a method by which the amount of influence exerted on the sentencing decision by each type of nonconformity may be made to correspond to the importance of that type [**5.2** and **7.3**]. Nonconformities will generally be classified by their degree of seriousness such as:

**Class A:** Those nonconformities considered to be of the highest concern for the product or service; such a class of nonconformity should be assigned a small AQL value.

**Class B:** Those nonconformities considered to be of the next lower degree of concern; therefore this class should be assigned a larger AQL value than class A and smaller than class C, and so on.

The number of classes and the assignment into a class should be appropriate to the quality requirements of the specific situation.

There are various ways of allocating AQLs to classes. Possibly the simplest is to group all the nonconformities into two classes A and B, and allocate a single AQL to each class, as in the following example.

| Class | AQL |
|---|---|
| A | 0,40 % nonconforming |
| B | 1,5 % nonconforming |

There would then be two separate sampling plans corresponding to these AQLs, and if a lot passed on each of the two plans it would be accepted; if it failed on either or both of them, it should be not accepted.

Alternative possibilities are as follows.

a) To have more than two classes but still sentencing each class separately, for example:

| Class | AQL |
|---|---|
| A | 0,65 % nonconforming |
| B | 1,5 % nonconforming |
| C | 4,0 % nonconforming |

b) To allocate a separate AQL to each feature, possibly with an overriding AQL in addition for all features taken together, or for all features in a class, for example an AQL of 1,0 for each of three nonconformities and an AQL of 1,5 for all of the three nonconformities. This method may be valuable where the article is complex and has many independent features to be inspected [**3.3**].

c) To consider class A by itself, but then aggregate all nonconformities to consider class A and class B together. AQLs might be set as, for example:

| Class | AQL |
|---|---|
| A | 1,0 % nonconforming |
| A + B | 4,0 % nonconforming |

Only the simplest method will be considered in detail in this part of ISO 2859. While the other methods undoubtedly have their place in appropriate circumstances, it has to be understood that the working of a complicated plan can become formidable to the shop-floor personnel. In most cases simplicity is to be preferred.

EXAMPLE 16

A product has five dimensions to be checked on each article inspected. On consideration of the effects of nonconformities of each type, it is decided that dimensions 1 and 2 have to be assigned to class A, whilst the other three dimensions may be grouped into class B.

Suppose the AQLs are chosen as:

| Class | AQL |
|---|---|
| A | 0,65 % nonconforming |
| B | 2,5 % nonconforming |

Suppose that for both classes the inspection level is III, and single sampling and normal inspection are to be used with batches of size 900. The sample size code letter is K. The following are the sample plans:

| Class | Sample size | Acceptance number (nonconforming items) | Rejection number (nonconforming items) |
|---|---|---|---|
| A | 125 | 2 | 3 |
| B | 125 | 7 | 8 |

The pattern, the same sample size for each class but different acceptance numbers, is typical and makes the administration of sampling easier, as the same physical sample may be used for each class (provided the inspection is not destructive for more than one of the types of nonconformity).

From a particular lot, a sample of 125 might give the following results:

— 1 item nonconforming in dimension 1 only,

— 1 item nonconforming in dimensions 2 and 4,

— 2 items nonconforming in dimension 3 only,

— 3 items nonconforming in dimensions 3 and 4.

There are 2 nonconforming items of class A and 5 of class B. The lot is acceptable.

EXAMPLE 17

A product is to be inspected under the following conditions: lot size 500, inspection level II, normal inspection, single sampling. The AQLs are:

| Class | AQL |
|---|---|
| A | 0,065 % nonconforming |
| B | 0,25 % nonconforming |

The sampling plans are found to be:

| Class | Sample size | Acceptance number (nonconforming items) | Rejection number (nonconforming items) |
|---|---|---|---|
| A | 200 | 0 | 1 |
| B | 50 | 0 | 1 |

In this situation, a sample of 50 should be examined for all types of nonconformity, and then a further sample of 150 for nonconformities of class A only.

Alternatively, as a sample of 200 is needed anyway, the inspector may decide that it would be as well to inspect a sample of this size for both classes. This also overcomes any psychological problems that could arise from an inspector having to ignore some class B nonconformities in the 150 items examined for class A only. He may do this, provided that the responsible authority approves [**3.10**]. By using code letter L, the plan for class B becomes:

| | |
|---|---|
| sample size | $n = 200$ |
| acceptance number | Ac = 1 |
| rejection number | Re = 2 |

When nonconformities are classified with separate AQLs for the different classes, then the switching between normal and tightened inspection is done independently for each class, or group of classes, for which an AQL is specified, according to the acceptances or non-acceptances for that particular class or group.

EXAMPLE 18

The conditions are: lot size 275, inspection level III, single sampling (sample size code letter H), AQLs of 1,5 % nonconforming for class A and 4,0 % nonconforming for class B.

Table 4 shows some hypothetical results and the manner in which the switching is done.

NOTE 5 So much switching in such a short experience is useful for the sake of an example but unlikely in real life.

## 3.4 Lots

Lot inspection is introduced in **2.4** where, in discussing lot size (see **2.4.2**), large lots are advocated provided all the items in a lot are produced under essentially the same conditions. Small lots which are likely to be of dissimilar quality should not be combined to make a larger inspection lot.

The formation of lots is discussed further in this subclause, with examples.

EXAMPLE 19

A producer is making articles to be inspected under the following conditions: AQL 2,5 % nonconforming, inspection level II, normal inspection, single sampling.

He has two machines, say A and B. Each machine produces 900 articles per hour, and it is decided that one hour's production from one machine is to be a lot. Reference to the tables for the conditions given above and a lot size of 900 gives the following sampling plan, under code letter J:

| | |
|---|---|
| sample size | $n$ = 80 items |
| acceptance number | Ac = 5 nonconforming items |
| rejection number | Re = 6 nonconforming items |

Its OC curve can be found in Table X-J (AQL = 2,5).

A suggestion is made that it might be advantageous to change the basis of aggregating to one hour's production from the two machines taken together, thereby increasing the lot size from 900 to 1 800. If this were done, the tables show that the sampling plan becomes, under code letter K:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 7 nonconforming items |
| rejection number | Re = 8 nonconforming items |

The new OC curve can be found in Table X-K (AQL = 2,5).

Whether this would be advantageous depends upon whether machines A and B are producing to the same quality or not. As a demonstration, consider the following three possible cases.

a) **Case 1**

Machines A and B are both producing to the same quality of 2,3 % nonconforming. This quality is better than the AQL, so it is desirable that the overwhelming majority of lots would be accepted by the sampling procedure.

If the lot size is 900, and the sample size is 80, the OC curve shows that the sampling plan will accept almost 99 % of the lots and just over 1 % will be not accepted. The number of articles inspected will be 160 per hour.

If the lot size is 1 800 and the sample is 125, the OC curve shows that just over 99 % of the lots will be accepted and just under 1 % not accepted. Only 125 articles will be inspected per hour.

The larger lot size is clearly better. All lots, both those accepted and those not accepted, have the same 2,3 % nonconforming level.

b) **Case 2**

Machines A and B are both producing to the same quality of 10 % nonconforming. This quality is worse than the AQL so it is desirable that most of the lots would be not accepted.

If the lot size is 900, and the sample size is 80, the OC curve shows that the lots will have only a 20 % chance of being accepted. The number of articles inspected will be 160 per hour.

If the lot size is 1 800, and the sample is 125, the OC curve shows that the lots will have only an 8 % chance of being accepted. The number of articles inspected will be 125 per hour.

Again the larger lot size is clearly better as all lots have the same 10 % nonconforming items in them.

c) **Case 3**

Machine A is producing a quality of 2,3 % nonconforming and machine B a quality of 10 % nonconforming.

If the lot size is 900 and the sample size is 80, the OC curve shows that about 99 % will be accepted and 1 % not accepted of the "A" lots, while 20 % of the "B" lots will be accepted and 80 % not accepted.

As a whole. therefore, (99 % + 20 %)/2 of lots will be accepted, i.e. about 60% of lots, and (1% + 80 %)/2 of lots will be not accepted, i.e. about 40 % of lots.

The accepted lots will have an average percent nonconforming level of

$$\frac{99}{99+20} \times 2{,}3 \text{ \% nonconforming}$$

$$+ \frac{20}{99+20} \times 10 \text{ \% nornconforming}$$

i.e. 3,6 % nonconforming. The number of articles inspected will be 160 per hour.

If the lot size is 1 800 and the sample size is 125, the quality of the lots will be 0,5 × (2,3 % nonconforming + 10 % nonconforming), i.e. 6,15 % nonconforming. The OC curve shows that 50 % of lots will be accepted and 50 % not accepted. The number of articles inspected will be 125 per hour.

The larger lot size means less inspection, as in cases 1 and 2, but there is a price to be paid. Instead of 60 % of the lots being accepted with an average quality of 3,6 %, nonconforming, 50 % of the lots are accepted and these are 6,15 % nonconforming.

**Table 4 — Sampling 20 lots from a hypothetical inspection process, inspection level III**
(see example 18)

| Lot number | Lot Size | Sample Size | Class A (AQL = 1,5 % nonconforming) | | | | | Class B (AQL = 4,0 % nonconforming) | | | | | Overall acceptability |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Ac | Re | Nonconforming item | Acceptability | Future action | Ac Re | | Nonconforming items | Acceptability | Future action | |
| 36 | 275 | 50 | 2 | 3 | 2 | A | Continue normal | 5 | 6 | 3 | A | Continue normal | A |
| 37 | 275 | 50 | 2 | 3 | 1 | A | Continue normal | 5 | 6 | 4 | A | Continue normal | A |
| 38 | 275 | 50 | 2 | 3 | 3 | N | Continue normal | 5 | 6 | 3 | A | Continue normal | N |
| 39 | 275 | 50 | 2 | 3 | 2 | A | Continue normal | 5 | 6 | 3 | A | Continue normal | A |
| 40 | 275 | 50 | 2 | 3 | 4 | N | Switch to tightened | 5 | 6 | 5 | A | Continue normal | N |
| 41 | 275 | 50 | 1 | 2 | 2 | N | Continue tightened | 5 | 6 | 4 | A | Continue normal | N |
| 42 | 275 | 50 | 1 | 2 | 3 | N | Continue tightened | 5 | 6 | 8 | N | Continue normal | N |
| 43 | 275 | 50 | 1 | 2 | 1 | A | Continue tightened | 5 | 6 | 6 | N | Switch to tightened | N |
| 44 | 275 | 50 | 1 | 2 | 1 | A | Continue tightened | 3 | 4 | 5 | N | Continue tightened | N |
| 45 | 275 | 50 | 1 | 2 | 0 | A | Continue tightened | 3 | 4 | 3 | A | Continue tightened | A |
| 46 | 275 | 50 | 1 | 2 | 0 | A | Continue tightened | 3 | 4 | 5 | N | Continue tightened | N |
| 47 | 275 | 50 | 1 | 2 | 1 | A | Restore normal | 3 | 4 | 2 | A | Continue tightened | A |
| 48 | 275 | 50 | 2 | 3 | 1 | A | Continue normal | 3 | 4 | 2 | A | Continue tightened | A |
| 49 | 275 | 50 | 2 | 3 | 1 | A | Continue normal | 3 | 4 | 1 | A | Continue tightened | A |
| 50 | 275 | 50 | 2 | 3 | 0 | A | Continue normal | 3 | 4 | 0 | A | Continue tightened | A |
| 51 | 275 | 50 | 2 | 3 | 1 | A | Continue normal | 3 | 4 | 2 | A | Restore normal | A |
| 52 | 275 | 50 | 2 | 3 | 1 | A | Continue normal | 5 | 6 | 2 | A | Continue normal | A |
| 53 | 275 | 50 | 2 | 3 | 0 | A | Continue normal | 5 | 6 | 1 | A | Continue normal | A |
| 54 | 275 | 50 | 2 | 3 | 2 | A | Continue normal | 5 | 6 | 4 | A | Continue normal | A |
| 55 | 275 | 50 | 2 | 3 | 2 | A | Continue normal | 5 | 6 | 3 | A | Continue normal | A |

A = acceptable N = not acceptable

In either case, of course, such a low acceptance rate would soon alert both the producer and the inspector to the fact that the production was not of the required quality and steps would be required to improve it. If the production from the two machines has been sentenced separately, it will be easy to see where the trouble is, but if the product has been mixed it may not be at all obvious that only one of the two machines is to blame.

This example is exaggerated, of course, in that the qualities from the two machines in case 3 (2,3 % nonconforming and 10 % nonconforming) are so very different. If they were closer in quality, the results of combining the lots would not be as serious, but the principle would be the same.

In practice, a lot very often contains items originating from more than one source. The sources may produce at different levels of quality and each source may not contribute an equal share of the total number of items making up the lot. Typical examples are parts from a multicavity mould, from a multispindle automatic lathe or from several similar production lines. The production may be so arranged that it is not easy to identify their separate sources without making special arrangements which could be inconvenient and costly; furthermore, it may be necessary to include the production from all such sources in order to make up lots of the required size.

The question may then be asked as to whether the OC curve for a sampling plan is still valid for lots such as these when they include items from a number of different sources which could be producing at different quality levels and are thus not strictly homogeneous.

The answer is that this makes no difference whatever to the validity of the OC curve, but it may lead to the non-acceptance of good product (because it has been mixed with bad product) or to the acceptance of bad product (because it has been mixed with good product), whereas, had they been kept separate, the good could have been accepted and the bad not accepted.

If, however, one or more sources have a quality level which is considerably worse than the others, then the effect will quickly show in the acceptance rate for the total and an investigation should be made. This should indicate the erring source and if this cannot be rectified it should be isolated and inspected separately.

## 3.5 Meaning of inspection level

The inspection level defines a relationship between lot size and sample size. The tables are planned so that when the lot size is large the sample is, in general, larger than when the lot size is small. It does not, however, get larger in proportion; for a large lot the sample is a smaller proportion than for a small lot.

Table I of ISO 2859-1 gives three general inspection levels numbered I, II and III, and four special inspection levels numbered S-1, S-2, S-3 and S-4. The general levels will be the most often used, and it is designated that level II will be used unless one of the other levels is specified [**10.1**].

Level I gives rather less than half the sample size of level II, whereas level III gives about one and a half times the sample size of level II.

EXAMPLE 20

For a lot size of 600, the sample sizes for the different inspection levels are:

| **Inspection level** | **Code letter** | **Sample size** (single sampling) |
|---|---|---|
| I | G | 32 |
| II | J | J |
| III | K | 125 |

It has to be remembered, however, that for certain AQLs the arrows in the table will lead to sample sizes different from these.

A complete table of sample size as a proportion of lot size would need to take into account the AQL also, because of the arrows. Even for a given value, the relationship is not a smooth one as only certain values of the sample size are available, whereas it is necessary to allow for all possible lot sizes. As a result, such a table would tend to confuse rather than to enlighten. However, a useful summary of the relationship is given in Table 5.

The special inspection levels are designed for situations where the sample size has to be kept small. They should not be specified without carefully examining the implications, in terms of sampling risks, by a study of the OC curve.

ISO 2859-1 states: “In the designation of inspection levels S-1 to S-4, care shall be exercised to avoid AQLs inconsistent with these inspection levels” [**10.1**]. The point is that the main purpose of the special inspection levels is to keep samples small if really necessary. For instance, the code letters under S-1 go no further than D, equivalent to a single sample of 8, but it is of no use to specify S-1, in the hope of keeping the sample size down to 8 or less, if the AQL is 0,10 for which the minimum sample size is 125.

**Table 5 — Relationship between sample size and lot size for the three general inspection levels**

| Sample size as percentage of lot size (single sampling normal inspection) | Level I Lot size | Level II Lot size | Level III Lot size |
|---|---|---|---|
| Not more than 50 % | At least 4 | At least 4 | At least 10 |
| Not more than 30 % | At least 7 | At least 27 | At least 167 |
| Not more than 20 % | At least 10 | At least 160 | At least 625 |
| Not more than 10 % | At least 50 | At least 1 250 | At least 2 000 |
| Not more than 5 % | At least 640 | At least 4 000 | At least 6 300 |
| Not more than 1% | At least 12 500 | At least 50 000 | At least 80 000 |

NOTE This table should be regarded as an indication only. The values of lot size quoted are such that all larger sizes meet the required condition. In most cases some smaller lot sizes also meet it, but in every case a lot size of one item less than that quoted above fails to meet it.

The amount of information about the process quality gained from examining samples depends upon the absolute size of the samples, not upon the percentage of the lot that is examined. It is sometimes asked, therefore: “Why is the sample size made to depend upon the lot size?”

There are three reasons, as follows.

a) When there is more at stake, it is more important to take the right decision. Proper use of the tables leads to the result that lots from a good process become more likely to be accepted as the lot size increases, whereas lots from a bad process conversely become more likely to be not accepted.

b) With a large lot a sample size can be afforded that would be uneconomic for a small lot; for example a sample size of 80 from a lot of 1 000 may be easy to justify economically, where a sample of 80 from a lot of 100 would be relatively expensive in that most of the lot is being inspected.

c) Correct random selection (see **2.25**) is more difficult to ensure if the sample is too small a proportion of the lot.

## 3.6 Setting an inspection level

When using ISO 2859-1 in the circumstances for which it was designed (a long sequence of lots), it is necessary, before the tables can be used, to set the values of the AQL and inspection level. Often, indeed, it will be necessary to set these values before production can start.

Having set the AQL, as the quality required as a process average (see **2.7**), the inspection level [**10.1**] should be set by considering what quality should have a high chance of non-acceptance if an occasional lot of that quality should be offered. An inspection level can then be sought that will give the OC curve required for this purpose when the lot size is within the limits usually expected.

When both the AQL and LQ are given, it is possible to discover an OC curve from the tables (Table X) which satisfies both conditions and hence obtain the appropriate sample size code letter.

An inspection level can then be found (from Table I) which gives this code letter for a range of lot sizes that contains the lot size expected to be produced.

EXAMPLE 21

An AQL of 1,5 % nonconforming has been chosen and it is desired to have at least an 80 % chance of not accepting a 6 % nonconforming lot if such a lot should be offered while normal inspection is in operation. Looking at the OC curves in the extended tables, it is found that code letters A to J, for an AQL of 1,5, all fail to meet the requirement. Code letter K almost meets it precisely (in fact, the chance of not accepting a 6 % nonconforming lot is slightly less than 80 %, but it is close enough for most practical purposes). Code letters L - P more than meet the requirement.

Suppose the lot size normally to be expected is 1 000. Then inspection level III can be specified, since this will give code letter K for a 1 000 lot size. If at a later stage the lot size is increased, the specified inspection level may call for code letters coming after K in the alphabet. This is satisfactory, as it means that the increased lot size is being put to good use in reducing the risks of accepting bad lots or not accepting good lots. From this point of view, there is no need to put an upper limit on the lot size (although there may be a need for such a limit for other reasons). A lower limit will be required, however, to ensure that code letters coming before K in the alphabet will not be used. For inspection level III, the lower lot size limit should not be less than 501 to ensure the use of code letter K.

EXAMPLE 22

An AQL of 0,40 % nonconforming has been chosen. For lots of 10 000, it is required to have at least 95 % chance of non-acceptance if a 1 % nonconforming lot should be offered while normal inspection is in effect.

Looking at the OC curves for an AQL of 0,40, it will be found that even code letter R does not meet the requirement. It should then be questioned whether this requirement is really essential. If it is decided that it is, then the only course is to tighten the AQL. When tightened to 0,25 % nonconforming, it is found that code letter R does meet the requirement.

However, none of the inspection levels in Table I gives code letter R for a lot of 10 000. It is necessary to specify code letter R as such, instead of specifying an inspection level.

It should be noted that the inspection levels given are not the only possible inspection levels, and it may sometimes be necessary to specify a "special" inspection level for a special occasion. A particular case of such a "special" level would be to give a constant code letter whatever the lot size if, for example, a definite fixed OC curve were required, as in example 21.

At the commencement of production, or when the records of past production are not available, it may be desirable to use 100 % inspection for a period to establish the quality capability of the production.

Alternatively, if a sampling procedure is to be used, it may be advisable to select the highest inspection level that is either practicable or economic for the initial production run. A change may then be made to a lower inspection level if the process average records indicate that the consumer's risk at this new level is acceptable. It should be noted that the choice of a lower inspection level increases the consumer's risk at the LQ to a greater extent than it affects the probability of acceptance when the submitted quality is equal to the AQL or better.

Another use for more than one inspection level occurs when the tables are being applied to the same product by two different inspection organizations, such as a main contractor and a sub-contractor or a producer and a government inspectorate. The same AQL should be used by both and applied to the same features, but the consumer may require the producer's inspector to use a higher inspection level than that being used for acceptance. Other sampling procedures are available for this type of situation but they are outside the scope of this part of ISO 2859.

It is also possible that a low inspection level may have to be used either for economic reasons or because the sampling tests are destructive. The inspector should then inspect the whole of each sample (avoiding curtailment) and periodically calculate the estimated process average. If the process average is then recorded in the form of a control chart (see, for example, ISO 8258:1991, *Shewhart control charts*), it will clearly show whether the quality requirements are being met. Although it may not then be possible to deal with past production, the information will be available to enable measures to be taken for making improvements in the future.

One of the objections to a low inspection level is that the limiting quality at, say, 10 % consumer's risk is high compared with the AQL. If, however, the records from a continuing series of lots are examined, it may be found that the cumulative sample is equivalent to one taken for a plan at a higher inspection level, and possibly to a code letter later in the alphabet, for which the consumer's risk at the LQ is much more acceptable. If then the cumulative results are compared with this new plan, the acceptance or non-acceptance decisions that have been made can be reviewed.

EXAMPLE 23

An external inspection organization is at present examining the output from two producers, A and B. It is proposed to apply sampling inspection using an AQL of 1% nonconforming in lieu of the present 100 % examination.

Producer A manufactures lots of approximately 4 000 items with a process average of 0,8 % nonconforming. Occasionally, lots are found which are up to 4 % nonconforming.

To assist in the selection of the inspection level, the OC curves for the general inspection levels I, II and III are studied (Figure 6). it is decided that more assurance than is provided by the level II plan ($n$ = 200, Ac = 5, Re = 6) is required to guard against the acceptance of lots containing 4 % nonconforming. Accordingly, level III is selected and the plan $n$ = 315, Ac = 7, Re = 8 is used. The change in probability of acceptance achieved, at 4 % nonconforming incoming quality, is from 19 %, using level II, to 7 % with the level III plan.

Producer B manufactures in similar size lots (approximately 3 500 items) but has a higher quality record. The true process average varies from 0,4 % to 1,7 % nonconforming.

From Figure 6 it is apparent that there is very little practical difference in the OC curves for code letters J, L and M given by Table I for inspection levels I, II and III, respectively, for incoming qualities of 1,7 % nonconforming. Inspection level I is therefore specified with a consequent saving in the number of sample items inspected. It would be advantageous if arrangements could be made to reward the producer for saving inspection costs.

## 3.7 Preferred AQLs

The tables in ISO 2859-1 give 26 AQLs ranging from 0,010 (i.e. 1 nonconforming item per 10 000 items of product) to 1 000 (i.e. 1 000 nonconformities per 100 items of product or an average of 10 nonconformities per item). These 26 AQLs are chosen so that each is approximately one and a half times as large as the previous one (the average ratio is in fact the fifth root of 10, or 1,585).

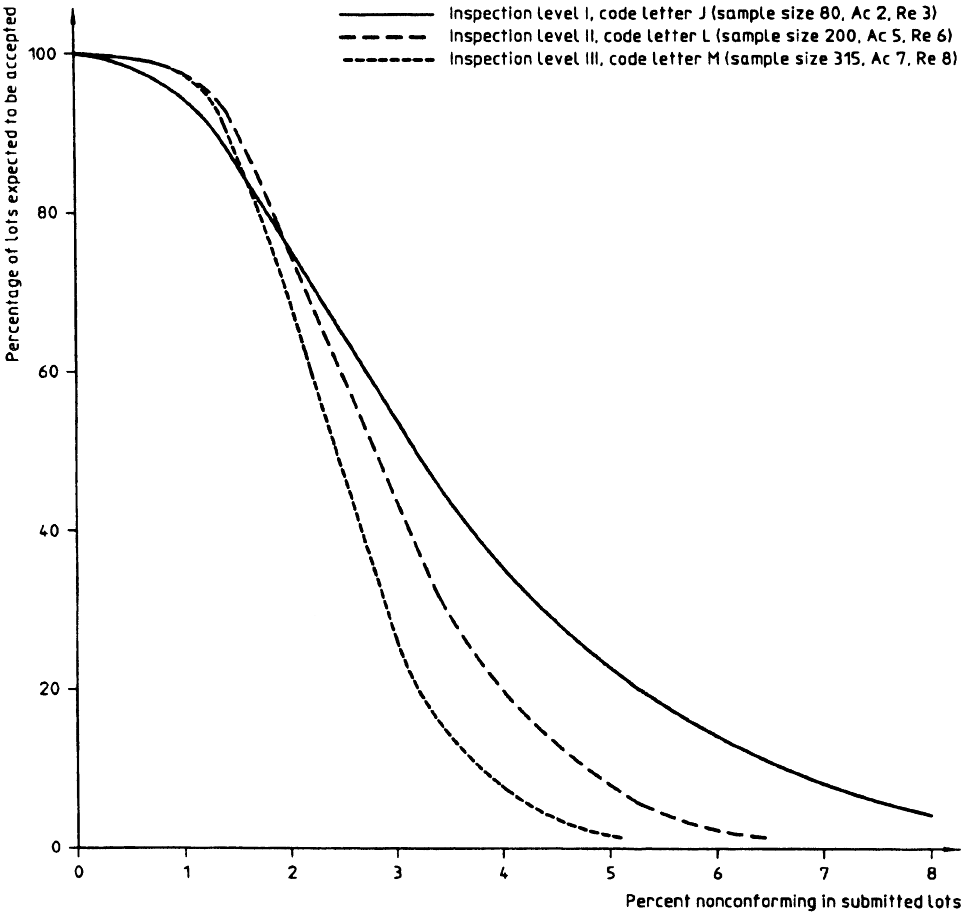



**Figure 6 — Comparison of OC curves for determining inspection level (AQL of 1% nonconforming, normal inspection)**

When the specified AQL for inspecting any given product is one of these preferred AQLs, the tables may be used. If, however, a specified AQL is not a preferred AQL, the tables are not applicable [**5.3**]. In these circumstances, reference should be made to whoever specified the AQL, with a request to have it examined to see whether a preferred value would be satisfactory. If not, then a sampling plan has to be designed specially for the particular AQL required (see **3.8**).

The very high values of AQL, 100 and above, are not likely to be used often, as they imply that a product in which every item contains nonconformities may be considered satisfactory. Clearly, this could be so only if the nonconformities being sought were of a minor nature, and the unit of product was something fairly complex, such as a complete vehicle.

EXAMPLE 24

In the inspection of cloth, to be made up later into clothing, the unit of product might be a considerable area of cloth. In the inspection for minor weaving faults, an average of 4 faults per square metre might well be acceptable, in which case an AQL of 400 nonconformities per 100 $m^2$ could be specified.

## 3.8 Non-preferred AQLs

For ease of administration, it is advisable to use preferred AQL values as much as possible. However, the pattern of ISO 2859-1 makes it an easy matter to design for other AQL values sampling plans that are consistent with the ISO 2859-1 scheme.

Table 6 gives a summary in which all sample size values and AQL values are expressed in terms of one variable, namely *n*, the sample size for single sampling. Choosing a given value of *n* at once gives a series of plans for that sample size and various AQLs, with the corresponding double and multiple plans.

**Table 6 — Summary table of ISO 2859-1 sampling plans for normal and tightened inspection**

| Type of sampling plan | Sample size | Cumulative sample size | Acceptable quality level (normal inspection) | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 12.5/*n* | | 50/*n* | | 80/*n* | | 125/*n* | | 200/*n* | | 315/*n* | | — | | 500/*n* | | — | | 800/*n* | | — | | 1 250/*n* | |
| | | | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re | Ac | Re |
| **Single** | *n* | *n* | 0 | 1 | 1 | 2 | 2 | 3 | 3 | 4 | 5 | 6 | 7 | 8 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 18 | 19 | 21 | 22 |
| Double | 0,63*n* | 0,63*n* | a | | 0 | 2 | 0 | 3 | 1 | 4 | 2 | 5 | 3 | 7 | 3 | 7 | 5 | 9 | 6 | 10 | 7 | 11 | 9 | 14 | 11 | 16 |
| | 0,63*n* | 1.26*n* | | | 1 | 2 | 3 | 4 | 4 | 5 | 6 | 7 | 8 | 9 | 11 | 12 | 12 | 13 | 15 | 16 | 18 | 19 | 23 | 24 | 26 | 27 |
| Multiple | 0,25*n* | 0,25*n* | a | | # | 2 | # | 2 | # | 3 | # | 4 | 0 | 4 | 0 | 4 | 0 | 5 | 0 | 6 | 1 | 7 | 1 | 8 | 2 | 9 |
| | 0,25*n* | 0,50*n* | | | # | 2 | 0 | 3 | 0 | 3 | 1 | 5 | 1 | 6 | 2 | 7 | 3 | 8 | 3 | 9 | 4 | 10 | 6 | 12 | 7 | 14 |
| | 0,25*n* | 0,75*n* | | | 0 | 2 | 0 | 3 | 1 | 4 | 2 | 6 | 3 | 8 | 4 | 9 | 6 | 10 | 7 | 12 | 8 | 13 | 11 | 17 | 13 | 19 |
| | 0,25*n* | *n* | | | 0 | 3 | 1 | 4 | 2 | 5 | 3 | 7 | 5 | 10 | 6 | 11 | 8 | 13 | 10 | 15 | 12 | 17 | 16 | 22 | 19 | 25 |
| | 0,25*n* | 1,25*n* | | | 1 | 3 | 2 | 4 | 3 | 6 | 5 | 8 | 7 | 11 | 9 | 12 | 11 | 15 | 14 | 17 | 17 | 20 | 22 | 25 | 25 | 29 |
| | 0,25*n* | 1,50*n* | | | 1 | 3 | 3 | 5 | 4 | 6 | 7 | 9 | 10 | 12 | 12 | 14 | 14 | 17 | 18 | 20 | 21 | 23 | 27 | 29 | 31 | 33 |
| | 0,25*n* | 1,75*n* | | | 2 | 3 | 4 | 5 | 6 | 7 | 9 | 10 | 13 | 14 | 14 | 15 | 18 | 19 | 21 | 22 | 25 | 26 | 32 | 33 | 37 | 38 |
| | | | 20/*n* | | 80/n | | 125/n | | 200/n | | 315/n | | – | | 500/*n* | | – | | 800/*n* | | – | | 1 250/n | | – | |
| | | | Acceptable quality level (tightened inspection) | | | | | | | | | | | | | | | | | | | | | | | |
| [a] Use single sampling. | | | | | | | | | | | | | | | | | | | | | | | | | | |

EXAMPLE 25

An AQL of 2 % nonconforming has been specified and a set of sampling plans for single sampling and both normal and tightened inspection is required. Looking at the column for acceptance number 0, an AQL value of 12,5/$n$ is found for normal inspection. Setting 12,5/$n$ equal to 2 (i.e. 2 % nonconforming) leads to $n$ = 6,25, which is rounded to the nearest integer to give 6.

Similarly, the column for acceptance number 1 gives an AQL of 50/$n$. Setting 50/$n$ equal to 2 leads to $n$ = 25, and the other columns can be similarly used to derive the plans as follows:

| Sample size | Ac | Re |
|---|---|---|
| 6 | 0 | 1 |
| 25 | 1 | 2 |
| 40 | 2 | 3 |
| 63 | 3 | 4 |
| 100 | 5 | 6 |
| 158 | 7 | 8 |
| 250 | 10 | 11 |
| 400 | 14 | 15 |
| 625 | 21 | 22 |

For tightened inspection, the bottom scale of AQLs should be used instead of the top scale to give:

| Sample size | Ac | Re |
|---|---|---|
| 10 | 0 | 1 |
| 40 | 1 | 2 |
| 63 | 2 | 3 |
| 100 | 3 | 4 |
| 158 | 5 | 6 |
| 250 | 8 | 9 |
| 400 | 12 | 13 |
| 625 | 18 | 19 |

EXAMPLE 26

Double and multiple plans are required corresponding to the plan with sample size 100 (Ac = 5, Re = 6) as found in example 25.

Looking at the column for an acceptance number of 5 and with $n$ = 100, quickly leads to:

| Sample size | Combined sample size | Ac | Re |
|---|---|---|---|
| 63 | 63 | 2 | 5 |
| 63 | 126 | 6 | 7 |

| Sample size | Combined sample size | Ac | Re |
|---|---|---|---|
| 25 | 25 | # | 4 |
| 25 | 50 | 1 | 5 |
| 25 | 75 | 2 | 6 |
| 25 | 100 | 3 | 7 |
| 25 | 125 | 5 | 8 |
| 25 | 150 | 7 | 9 |
| 25 | 175 | 9 | 10 |

However, this approach does not provide the supplementary information, such as limit numbers for reduced inspection, so departure from preferred AQLs should not be a regular practice.

## 3.9 Setting an AQL

Subclause **2.6** introduces the concept of AQL, the consumer's and the producer's view of the AQL in relation to their requirements, and the effect that usage of the item and the consequence of failure will have in setting an AQL.

It is also necessary to consider how many components will be contained in the eventual equipment. If, for example, it is decided that the quality of a piece of equipment containing three independent and equally important components should be not more than 10 % nonconforming, then each of the three components could be 3,5 % nonconforming and the requirement would be met, whereas if it were to contain ten components, these would have to be no worse than 1% nonconforming.

Provided that the components conform or do not conform independently, the formula here is that if $k$ is the number of components in the assembly, $X$ is the AQL of the assembly, and $x$ is the AQL of the components, then from the multiplication law of probabilities, it follows that

$$\frac{X}{100} = 1 - \left( \frac{100 - x}{100} \right)^k$$

The value of $X$, however, does not take into account nonconformities which may arise through a faulty assembly process.

In these circumstances the producer would probably wish to choose what would seem to be a suitable AQL for each component and then calculate what quality can be expected in the overall equipment, whereas the consumer would wish to specify an AQL for the overall equipment and then calculate what has to be the quality for the components. In general, the second of these approaches is probably the more reasonable in that it is the performance of the overall equipment that really matters, but it is also the more expensive approach because it almost always leads to tighter AQLs. However, it has to be accepted that good quality in a complicated article is inevitably more expensive than equally good quality in a simple one.

The question of what quality level can reasonably be expected, at the price the consumer is prepared to pay, and with the methods of production envisaged, can often be answered by an examination of what quality level has been produced and tolerated in the past. Where the article is a new one, and there has been no past production, there will often be other similar articles from which relevant information can be obtained. Past process average calculations may be particularly helpful. This idea of looking at the quality obtained in the past should not be taken as meaning that past quality levels are sacrosanct and are always good enough. This is because the production cost of a nonconforming item is nearly equal to that of a conforming item and reduction of percent nonconforming frequently means reduction of production cost. It is simply one of the factors to be taken into account in assessing what is a reasonable AQL to set.

It has to be remembered that the mere setting of an AQL does not give the consumer a guarantee that lots of a worse quality will not be accepted. In the first place, the AQL refers to the average. Some lots may be worse than the AQL while the average is better than the AQL. In the second place, if the average quality being offered is a little worse than the AQL, a number of lots will probably be accepted before a switch to tightened inspection is called for, and even after the switch there is likely still to be some acceptance. In general, however, it can be expected that the consumer will get a product with an average which is better than the AQL, as sampling schemes have a built-in economic incentive in that a producer cannot afford to have more than a small proportion of lots not accepted and will take steps to improve the quality of production if this proportion is exceeded.

It might be thought that this is not very satisfactory from the consumer's point of view, relying as it does upon what is likely to happen rather than upon what will certainly happen. But in practice most manufacturers take steps to see that their process average does not exceed the AQL, if only because relatively frequent lot non-acceptance makes life difficult. In any case, the consumer's protection depends upon the lower end of the OC curve as well as upon the upper end with which the AQL is concerned, and this lower end can be adjusted by considering the LQ values of any suggested plan.

It is not necessary that the AQL should always be the primary choice from which all else is derived. It is always possible when circumstances so require to enter sampling tables "through the back door", choosing a plan by some other criterion and then finding the AQL specified to get the desired result. In this case, the AQL is a convenient index to enable the standard tables to be used, and is also valuable as an answer to the question in which the producer is primarily interested: At what quality has he to manufacture to get most lots accepted?

If such a "back-door" method is used, the primary choice may be either a low point on the curve, where this is thought particularly important, or some economic criterion. Probably the simplest economic criterion that has been suggested is to make an estimate of the break-even point; i.e. the lot quality such that, if the lot were accepted, the cost of the damage done by the accepted nonconforming items would be exactly the same as the cost of failing to accept the lot.

If this break-even quality can be estimated, it would be well to choose a plan for which this quality gives 50 % of lots expected to be accepted, not because a 50 % chance of acceptance with this quality is particularly wanted (by definition, if this particular quality is offered, it is not of much interest what the inspection plan does with it), but because this ensures a greater than 50 % chance of acceptance for better quality than break-even quality, and a greater than 50 % chance of non-acceptance for worse quality than break-even quality.

Finally, having taken all these factors into account, it is desirable to choose one of the AQL values given in the tables to be used if possible, as the tables are otherwise inapplicable and a special plan would have to be designed.

Some additional information is given in ISO/TR 8550.

### 3.10 Drawing a sampling plan from the tables in ISO 2859-1

Before drawing a sampling plan from the tables, it is necessary to know five things, as follows.

a) The acceptable quality level, or AQL (see **2.6**).

b) The inspection level (see **3.5**). In general, a) and b) will be specified for a particular product at the beginning of a contract and will remain constant throughout the run of that contract.

c) Whether normal, tightened or reduced inspection is to be used. This is decided by studying the sampling results of the last few lots, which is explained in detail in to **3.11** to **3.14**. For the moment, it will be assumed that normal inspection is being used.

d) Whether single, double or multiple sampling is to be used. For the moment, single sampling is assumed.

e) The lot size.

EXAMPLE 27

Suppose the AQL is 1,0, the inspection level is II and the lot size is 2 500.

The first thing required is the sample size code letter (usually called simply the code letter, for short). For a lot size of 2 500 and inspection level II, Table I gives the code letter as K.

In the appropriate master table (Table II-A), it is found that the sample size for single sampling is 125.

AQLs for normal inspection are given along the top of the table, and under the value 1,0 the numbers 3 and 4 are given under the heading Ac Re.

The sampling plan required is:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 3 |
| rejection number | Re = 4 |

Alternatively, the extended Table X-K-2 could be used.

Again the sample size of 125 is found; in the column for AQL = 1,0 are found the acceptance and rejection numbers 3 and 4 as before.

EXAMPLE 28

Suppose the AQL is 0,40, the inspection level is I and the lot size is 230. Table I gives the code letter as E.

Using the master Table II-A, it is found that there is no plan for letter E and AQL 0,40, but a downward-pointing arrow directs the user to letter G instead, and the required plan is:

| | |
|---|---|
| sample size | $n$ = 32 items |
| acceptance number | Ac = 0 |
| rejection number | Re = 1 |

Alternatively, in the extended Tables, X-E-2 is appropriate but this page has no column for AQL = 0,40. Instead, the symbol of an inverted triangle appears for AQLs less than 1,0. This triangle refers to the footnote "Use next subsequent sample size code letter for which acceptance and rejection numbers are available." If the triangle is thought of as an arrowhead, it is pointing in the direction in which the appropriate table will be found. This leads to letter F where again AQL = 0,40 is not given, and so on to letter G to find the same plan as before.

It is very important to remember that if a triangle or series of triangles directs the user from one page to another of the extended tables, or an arrow directs the user from one row to another of the master tables, the sample size to be used is the one given for the new page or the new row arrived at and not the one given for the original page or row [**10.3**].

Where upward-pointing arrows or triangles are found, the meaning is similar. They refer to the case where high values of the AQL are not given in the extended tables for the code letter indicated in Table I, and the symbol indicates "Use the next preceding sample size for which acceptance and rejection numbers are available." The triangles again point to the direction in which the appropriate table will be found.

EXAMPLE 29

Suppose the AQL is 0,015, the inspection level is III and the batch size is 120. Table I gives the code letter as G, but referring to the tables an arrow (or a series of triangles) leads to letter P before a plan is found. The required plan has a sample size of 800 which exceeds the lot size.

In this case the entire lot of 120 has to be taken as the sample. The acceptance and rejection numbers remain as 0 and 1.

ISO 2859-1 states that AQL values of 10 or less may express either percent nonconforming or nonconformities per 100 items, whereas values over 10 express only nonconformities per 100 items [**3.5**]. A decision has to be taken as to whether nonconformities or nonconforming items are appropriate in each particular case. The AQL should then be defined in terms of that decision. For this reason, examples 26, 27 and 28 are incomplete, for the AQL values are given as numbers without qualification, as are the acceptance and rejection numbers, which would be meaningless in practice. These examples are only given to show how a sampling plan may be taken from the tables.

EXAMPLE 30

In example 27, the AQL is 1,0 and the sampling plan is:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 3 |
| rejection number | Re = 4 |

The AQL needs to be defined, however, in terms of percent of nonconforming items, or of number of nonconformities per 100 items.

If the AQL were 1,0 % nonconforming, the sampling plan would be:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 3 nonconforming items |
| rejection number | Re = 4 nonconforming items |

If the AQL were 1,0 nonconformities per 100 items, the sampling plan would be:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 3 nonconformities |
| rejection number | Re = 4 nonconformities |

The tables, it will be seen, are used in precisely the same manner in either case.

## 3.11 Normal inspection

An AQL, it will be remembered, is the borderline in the quality scale between the good and the bad from an acceptance sampling viewpoint. When the AQL has been specified for any particular product, the ideal would be to have a system whereby lots could be always accepted when their quality was better than the AQL and never accepted when worse than the AQL; i.e. an OC curve which descended vertically at the AQL as shown in Figure 7. This ideal, however, is something that no sampling plan can produce, so an OC curve has to be accepted that descends at an angle less than the vertical.

Now, an OC curve can cross the ideal vertical line at only one point, and the question is: “At what point should it cross?”

One possible solution is to let the curve cross the vertical line near the bottom of the diagram, as in Figure 8. To choose a sampling plan that does this has the advantage of protecting the consumer, as, if any lot is submitted with a quality worse than the AQL, it will have a high chance of non-acceptance. Such a solution, however, is unsatisfactory from the producer’s point of view. He can have no complaint that, if he submits quality worse than the AQL, most of his lots will not be accepted, but he has a valid complaint if he submits quality better than the AQL and much of it is not accepted. In the case illustrated in Figure 8, just over one lot in five would be accepted if the rate of nonconforming items were only half the AQL, and less than half the lots would be accepted even if the rate of nonconforming items were as little as a quarter of the AQL. This is clearly unsatisfactory, as, if constant non-acceptances are to be avoided, it forces the producer to work to a considerably better quality than is really needed. This is likely to lead to production difficulties, and will seriously increase the price of the product; it is also likely to lead to bad relations between the producer and the inspection authority.

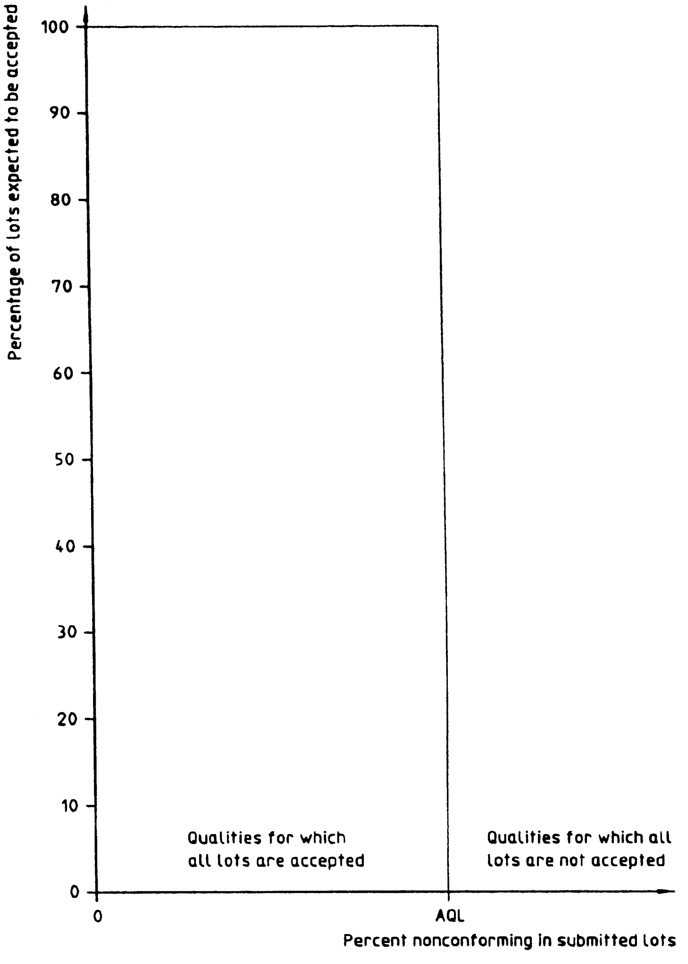



**Figure 7 — The ideal, but unattainable, OC curve**

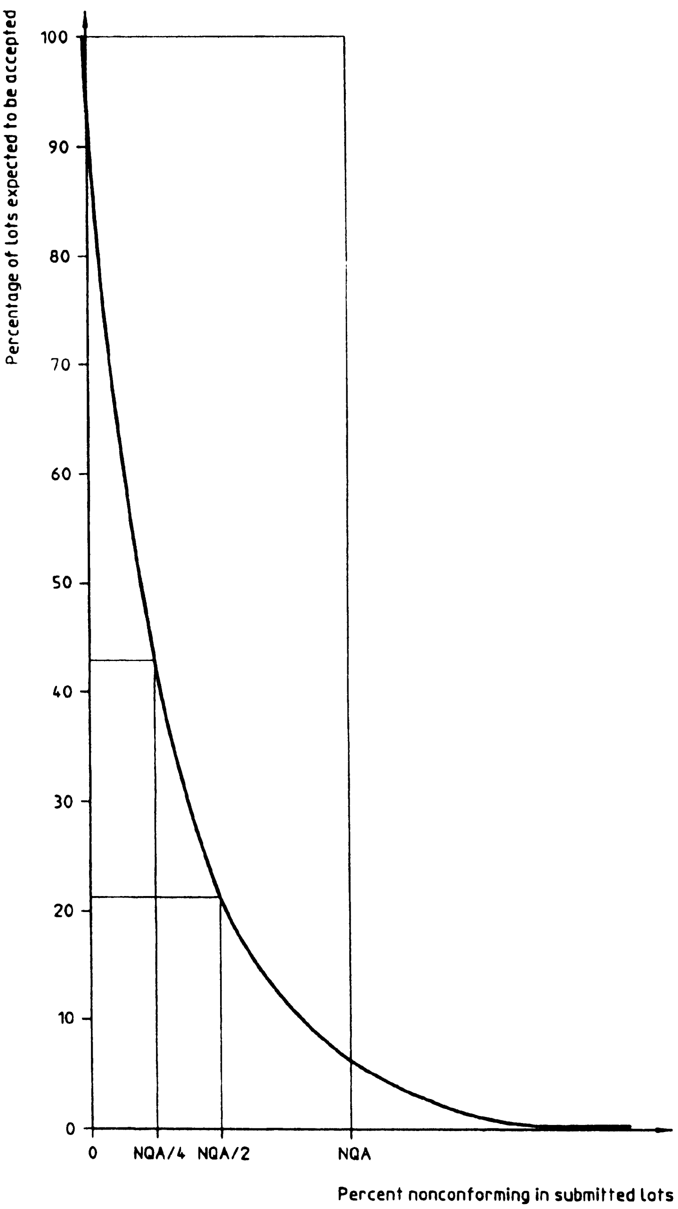



**Figure 8 — OC curve of a sampling plan designed to give a high probability of non-acceptance if any lot having a quality worse than the AQL is submitted**

An alternative solution, then, is to let the curve cross the vertical line near the top of the diagram, as in Figure 9. This will satisfy the producer, as, if he produces lots as good as, or better than, the AQL, they are almost certain to be accepted. It will now, however, be the consumer's turn to complain, for if the producer were to submit lots of a quality worse than the AQL, there might be a high probability of their being accepted. In the case illustrated in Figure 9 for instance, if lots were offered with a rate nonconforming equal to twice the AQL, then nearly 60 % of such lots would be accepted.

Where the sample size is relatively small for the required AQL, however, such a high assurance for the producer would entail too much risk for the consumer.

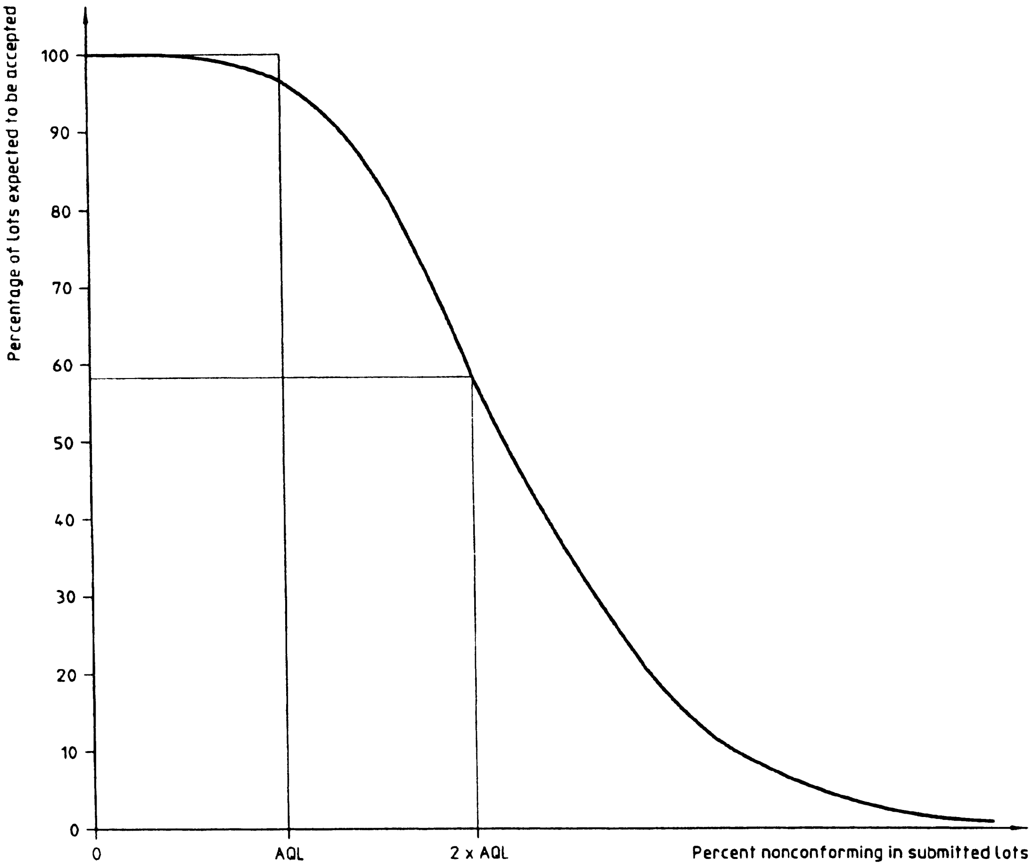



**Figure 9 — OC curve of a sampling plan designed to give a high probability of acceptance if any lot having a quality better than the AQL is submitted**

For the small acceptance numbers, therefore, a lower probability of acceptance at the AQL should be accepted. Figure 10 shows why this is so. It gives the OC curves for an AQL of 1 % nonconforming with the smallest and largest sample sizes given for this AQL. The producer is given much better assurance with the large sample size than with the small one if quality is good, but the curve slopes down so much more steeply that the consumer is also given better protection.

Normal inspection is designed, like the example in Figure 9, to protect the producer against having a high proportion of lots not accepted when quality is better than the AQL. Normal inspection, then, has OC curves that cross the vertical line at the AQL near the top, but the exact level at which they cross varies from plan to plan according to the value of the "AQL times sample size", or, what amounts to the same thing, according to the value of the acceptance number.

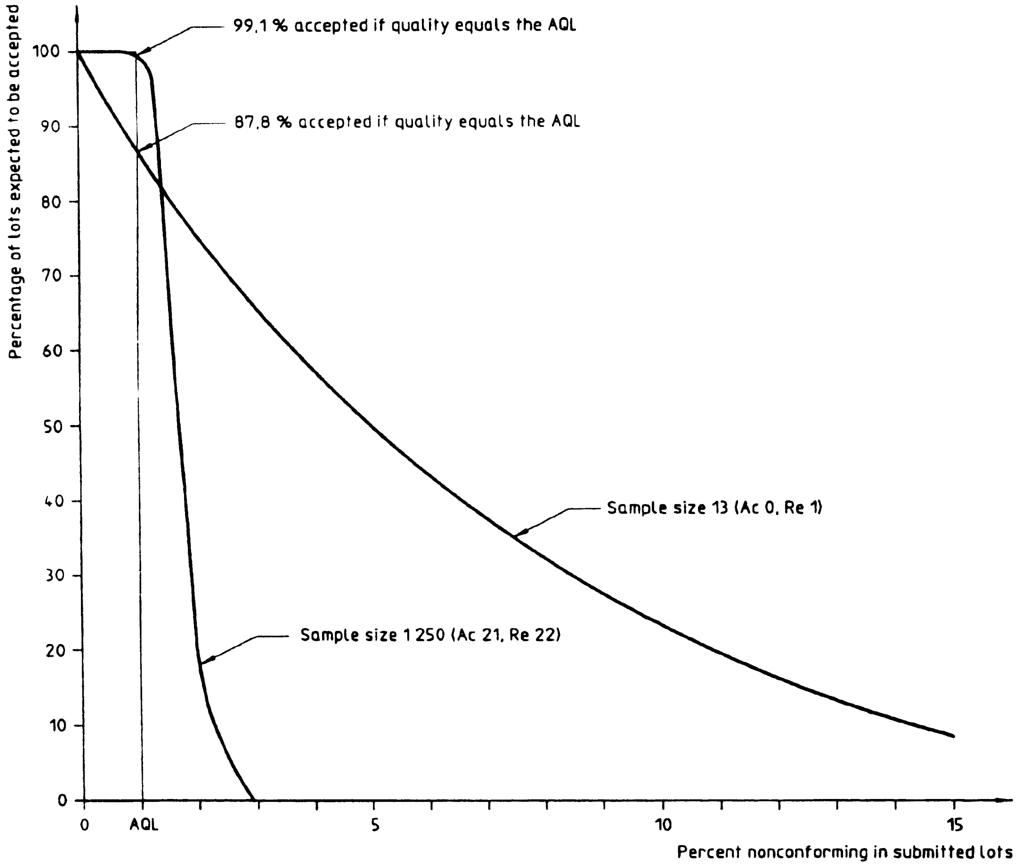



**Figure 10 — OC curves for two "normal inspection" sampling plans for an AQL of 1,0 % nonconforming**

Referring to Table 7, it can be seen that if the sample size is fairly large for the given AQL, leading to a value of "AQL times sample size" of at least 200, then the producer is always given at least 98 % assurance of the acceptance of his lot if the quality is equal to the AQL, and the assurance will be still higher for quality better than the AQL.

## 3.12 Tightened inspection

When tightened inspection is called for, the required plan is drawn from the tables in just the same way, except that Table II-B is used instead of Table II-A if the master tables are used, whereas if the extended tables (Table X) are used, the appropriate column of the table is found by reading the AQL value from the bottom of the table instead of from the top.

In general, it will be found that a tightened plan has the same sample size as the corresponding normal plan but a smaller acceptance number. However, if the normal inspection acceptance number is 1, changing to 0 would lead to an unreasonable degree of tightening, and if the normal inspection acceptance number is 0, no smaller number is available. In both these cases, tightening is achieved by keeping the acceptance number the same as for normal inspection while increasing the sample size.

OC curves for tightened inspection are not shown graphically so as to avoid confusing the diagrams by trying to put too much into them. However, the values of the probabilities of acceptance are given in Tables X, and where a plan exists both as a normal plan for one AQL and as a tightened plan for a different AQL, which is often the case, the same OC curve applies to the plan in both its guises. It has to be remembered that the figures used to label the curves refer to the normal inspection AQL values.

EXAMPLE 31

Suppose the AQL is 1,0, the inspection level is II and the lot size is 2 500.

From Table I the code letter is K. Using Table X-K-2, the tightened plan is:

| | |
|---|---|
| sample size | $n$ = 125 items |
| acceptance number | Ac = 2 nonconforming items |
| rejection number | Re = 3 nonconforming items |

This is the same as the normal plan for code letter K and an AQL of 0,65. Its OC curve is therefore the one labelled 0,65 in chart K.

**Table 7 — Percentage of lots expected to be accepted if quality is equal to AQL, single sampling, normal inspection**

| AQL × sample size (approx.) | Acceptance number | Percentage of lots expected to be accepted (approx.) |
|---|---|---|
| 12,5 | 0 | 88,1 |
| 50 | 1 | 90,9 |
| 80 | 2 | 95,3 |
| 125 | 3 | 96,1 |
| 200 | 5 | 98,3 |
| 315 | 7 | 98,4 |
| 500 | 10 | 98,5 |
| 800 | 14 | 98,3 |
| 1 250 | 21 | 99,0 |
| 2 000 | 30 | 98,7 |
| 3 150 | 44 | 98,5 |

NOTE The figures in the first column are approximate as it is impossible to make the values of “AQL × sample size” exactly constant on diagonals of Table II-A. As a result, the figures in the third column are inevitably approximate also, but it will be found that the true figure is always very close to the approximation given here.

## 3.13 Switching rules — Example

Subclause **2.9** introduced normal and tightened inspection and the last two subclauses have further discussed normal inspection and tightened inspection, what each is designed to do, and how to use the tables to find the appropriate sampling plans. Subclause **2.11** discusses the switching rules by means of which the decision is taken to change from normal to tightened or back again [**9.3**], or alternatively, to discontinue if the cumulative number of lots not accepted on original tightened inspection reaches 5 [**9.4**]. This subclause gives an example illustrating the operation of the switching rules when using ISO 2859-1.

EXAMPLE 32

A product is being supplied in lots of 4 000. The AQL is 1,5 % nonconforming. The inspection level is III. Single sampling is being employed. Table I gives the code letter as M, and the required sampling plans are found to be:

| | **Normal inspection** | **Tightened inspection** |
|---|---|---|
| sample size | 315 | 315 |
| acceptance number | 10 | 8 |
| rejection number | 11 | 9 |

Table 8 shows the results of the inspection of the first 25 lots. It is usual to use normal inspection at the start of a production run and this is done here. The nonacceptance at lots 4 and 10 do not cause a switch to tightened inspection, as in each case the 2 in 5 rule has not been met, but the non-acceptance at lot 12, following the one at lot 10, causes a switch for lot 13 onwards. At lot 21, five successive lots have been accepted on tightened inspection and normal inspection is restored as from lot 22.

**Table 8 — Twenty-five lots from a hypothetical inspection process**

| Lot number | Lot size | Sample size | Ac | Re | Nonconforming items | Acceptability | Future action |
|---|---|---|---|---|---|---|---|
| 1 | 4 000 | 315 | 10 | 11 | 7 | A | Continue normal |
| 2 | 4 000 | 315 | 10 | 11 | 2 | A | Continue normal |
| 3 | 4 000 | 315 | 10 | 11 | 4 | A | Continue normal |
| 4 | 4 000 | 315 | 10 | 11 | 11 | N | Continue normal |
| 5 | 4 000 | 315 | 10 | 11 | 9 | A | Continue normal |
| 6 | 4 000 | 315 | 10 | 11 | 4 | A | Continue normal |
| 7 | 4 000 | 315 | 10 | 11 | 7 | A | Continue normal |
| 8 | 4 000 | 315 | 10 | 11 | 3 | A | Continue normal |
| 9 | 4 000 | 315 | 10 | 11 | 2 | A | Continue normal |
| 10 | 4 000 | 315 | 10 | 11 | 12 | N | Continue normal |
| 11 | 4 000 | 315 | 10 | 11 | 8 | A | Continue normal |
| 12 | 4 000 | 315 | 10 | 11 | 11 | N | Switch to tightened |
| 13 | 4 000 | 315 | 8 | 9 | 7 | A | Continue tightened |
| 14 | 4 000 | 315 | 8 | 9 | 8 | A | Continue tightened |
| 15 | 4 000 | 315 | 8 | 9 | 4 | A | Continue tightened |
| 16 | 4 000 | 315 | 8 | 9 | 9 | N | Continue tightened |
| 17 | 4 000 | 315 | 8 | 9 | 3 | A | Continue tightened |
| 18 | 4 000 | 315 | 8 | 9 | 5 | A | Continue tightened |
| 19 | 4 000 | 315 | 8 | 9 | 2 | A | Continue tightened |
| 20 | 4 000 | 315 | 8 | 9 | 7 | A | Continue tightened |
| 21 | 4 000 | 315 | 8 | 9 | 6 | A | Restore normal |
| 22 | 4 000 | 315 | 10 | 11 | 7 | A | Continue normal |
| 23 | 4 000 | 315 | 10 | 11 | 2 | A | Continue normal |
| 24 | 4 000 | 315 | 10 | 11 | 5 | A | Continue normal |
| 25 | 4 000 | 315 | 10 | 11 | 3 | A | Continue normal |
| AQL = 1,5 % nonconforming, inspection level III (see example 32) | | | | | | | |
| A = acceptable N = not acceptable | | | | | | | |

## 3.14 Methods for reducing the sampling risks

There always have to be risks in sampling inspection, both of the acceptance of bad lots and of the nonacceptance of good lots, but these risks should be tolerable, provided that the AQL and inspection level have been well chosen.

If either the producer or the consumer should consider in a particular instance that the risk he is taking is too high, it would be well to check that the AQL and the inspection level have been well chosen, but for the remainder of this subclause it will be assumed that they are appropriate and were properly selected.

The producer will be interested in reducing risks when quality is better than the AQL (he is not entitled to any reduction of risk otherwise). The consumer will be particularly interested in the risks when quality is worse than the AQL, as, if quality is better than the AQL, he is getting the quality required.

There are three methods that can be used to reduce the risks for both parties.

The first method is to improve the quality of production. This may seem too obvious to be worth saying, but it is surprisingly easy in discussions on sampling plans, OC curves, switching rules, etc., to forget the simple rule that a low percentage nonconforming in the production gives the consumer what he wants and ensures a high proportion of acceptance to the producer.

The second method applies only in a particular case, but it is the case which is most likely to cause anxiety, namely, where the acceptance number is 0. Plans with a zero acceptance number have such shallow OC curves that big risks are unavoidable.

For this reason, ISO 2859-1 allows the use of an alternative when the tables lead to a zero acceptance number (provided the responsible authority approves). This alternative is to use the plan for the same AQL, with an acceptance number of 1 instead of 0 [**10.3**]. There is a price to be paid, in that a sample size about four times as big is required, but the risks for both parties are so much reduced that it is often well worthwhile.

The cost may be reduced somewhat by adopting double or multiple sampling (**2.20** and **2.21**). These alternatives become available when the acceptance number is 1 or more. Sequential sampling is a further possibility, but it is outside the scope of this section.

The third method is to consider the possibility of increasing the lot size. If the lot size can be increased far enough to lead to a change of code letter and an increase of sample size, this will reduce the risks for both parties, as the larger sample size leads to a steeper OC curve, and the tables are so arranged that this curve will be higher than the old curve at most points where quality is better than the AQL, and lower at most points where quality is worse than the AQL.

It is, unfortunately, not possible to arrange the tables so that these features are always as desired, without losing other desirable features. Figure 11 shows, as an example, four of the normal inspection plans associated with an AQL of 1,5 %. For quality better than the AQL, it is seen that the larger the sample the higher is the proportion of lots accepted, whereas for quality worse than twice the AQL, the largest sample does not accept most and the smallest sample does not accept least (and it is desired that the plan as often as possible does not accept when quality is worse than the AQL). The crossing point of the curves for sample sizes 32 and 50 is not so satisfactory.

The idea of increasing lot sizes, to gain better protection in the sampling, may be objected to as it is not always easy, or sensible, to change lot sizes. Lot sizes have to be fixed according to such things as continuity of production, the quantity of production that can be handled at one time, transport problems, stock control problems and so on. This is all true, but it is never-theless worth remembering that, other things being equal, an increased lot size can be helpful from the sampling inspection point of view. Refer to **3.4** for further discussion on lot size.

In examining the height of the curves in Figure 11 at twice, three times and four times the AQL, it has to be remembered that the curves show only part of the picture (the normal inspection part). For nearly all the normal inspection plans in ISO 2859-1, the percentage of lots accepted, if quality is twice the AQL, is less than 80 %. Such an acceptance rate will always lead to tightened inspection before long.

In some circumstances, it may be decided that the compromise necessarily involved in using a complete sampling scheme is not worthwhile. The parties involved may then negotiate to choose a plan direct from the OC curves, but where such an approach is adopted, the parties need to be knowledgeable if a satisfactory choice is to result.

## 3.15 Reduced inspection

Sometimes there is evidence that the production quality is consistently better than the AQL. Where this happens and there is reason to suppose that the good production will continue, a sampling plan is no longer needed to segregate the good lots from the bad ones, as all the lots will be good ones. Inspection cannot, however, be dispensed with altogether as a warning is needed if the production quality worsens.

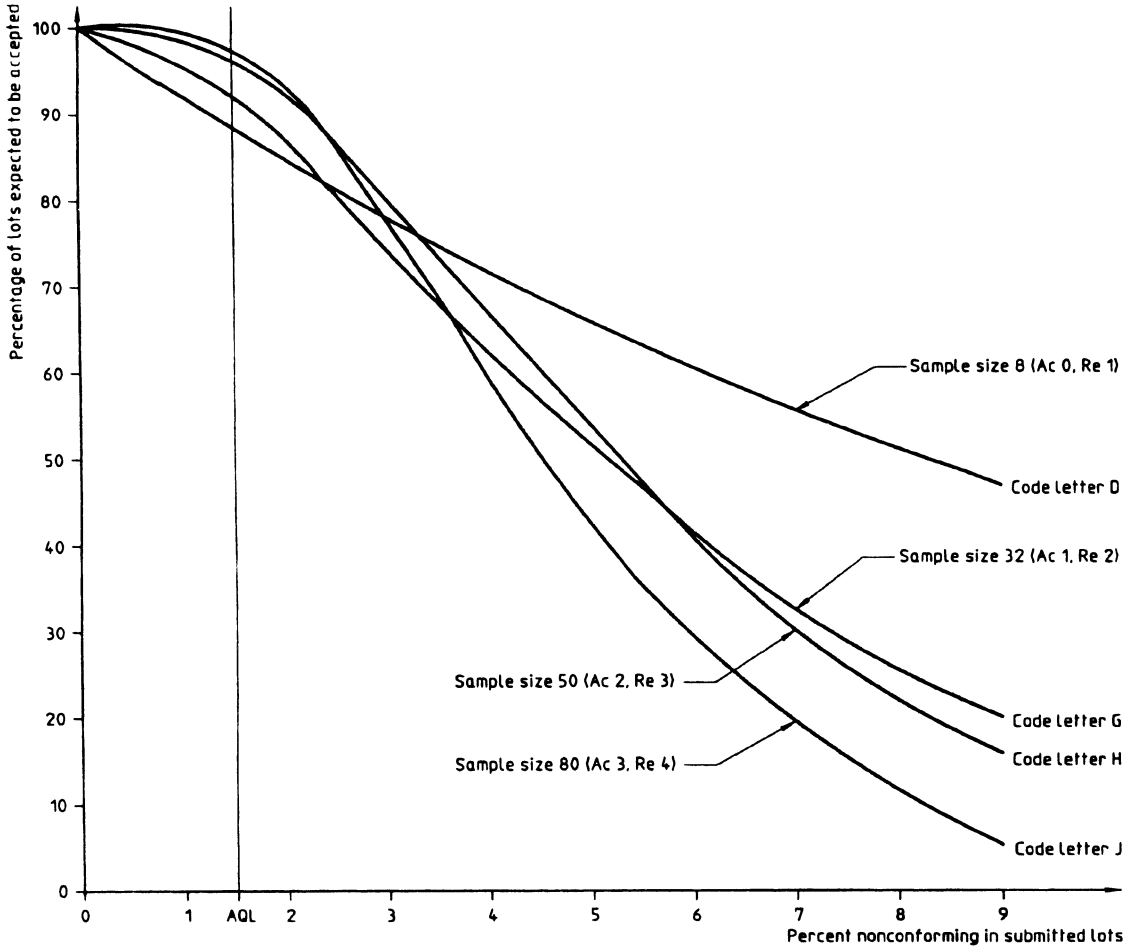



**Figure 11 — Four sampling plans for an AQL of 1,5 % nonconforming, normal inspection, single sampling**

In these circumstances, considerable savings can be made if so desired by using reduced-inspection sampling plans, which have sample sizes only two-fifths the size of the corresponding normal inspection plans (except where the normal inspection plan has a sample size less than 5, where the proportion is more than two-fifths, as reduced inspection always takes a sample of at least 2).

It might at first be thought that the way to reduce the sample size would be to use a code letter earlier in the alphabet. This would indeed reduce the sample size, but could have the undesirable effect of also reducing the proportion of lots expected to be accepted at the AQL; this would, in effect, mean penalizing the producer for doing good work. As such a result would clearly be unsatisfactory, a special table is necessary for reduced inspection. This is Table II-C of the master tables.

It should be noted that there is no compulsion about the introduction of reduced inspection. The use of tightened inspection, when called for by the switching rules, is essential to the scheme, but reduced inspection is entirely optional; even if the necessary switching conditions are met, a consumer need not introduce it unless he wishes to or the contract specifies its use.

The switching rules [**9.3.3**] are designed to ensure that reduced inspection is not introduced unless the observed quality is genuinely good and is likely to continue so. To detect whether reduced inspection is permissible, the recent production history has to be compared with a limit number, taken from Table VIII.

EXAMPLE 33

A product is being manufactured, to be inspected under the following conditions: an AQL of 10 % nonconforming, a lot size of 4 000 items, an inspection level of I with single sampling.

The normal plan is found under code letter J

sample size $n$ = 80 items

acceptance number Ac = 14 nonconforming items

rejection number Re = 15 nonconforming items

Table 9 shows the imaginary results of the inspection process. Normal inspection is in use at the beginning of the table, which is taken to be an extract from a longer sequence, so the lot numbers do not start at 1. The results are good, all lots being accepted, with the number of nonconforming items in each sample well below the acceptance number. After inspection of the sample from lot 51, the inspector decides to ask whether reduced inspection would be permissible. He counts the total number of nonconforming items observed in the samples from the last 10 lots and finds it to be 70. The number of sample items from the last 10 lots is 800, and looking against 800 and an AQL of 10 in Table VIII, the limit number is found to be 68; 70 is too many and reduced inspection is not permissible.

After very good results from the next 4 lots, he decides to try again after lot 55. The observed number of nonconforming items from the last 10 lots is now only 54, which is well within the limit number. Reduced inspection is now permissible provided that the previous 10 lots have all been accepted on normal inspection (which is the case) and provided that production is at a steady rate.

Just what is meant by “a steady rate” calls for some interpretation and this may well vary from one industry to another. The requirement is that there should have been no break in production sufficient to invalidate the argument that the present quality is almost certainly good, because the record of the recent past is so good. The precise meaning in any particular case has to depend upon technical judgement based upon the consideration of all the factors the variation of which can affect the quality of the product.

As reduced inspection is optional, the restoration of normal inspection is allowed if desired, and should be made if production becomes irregular or delayed, or if other conditions make it seem necessary. A return to normal inspection is required if a lot is not accepted on reduced inspection.

**Table 9 — Fifteen lots from a hypothetical inspection process**

| Lot number | Lot size | Sample size | Ac | Re | Nonconforming items | Acceptability | Future action |
|---|---|---|---|---|---|---|---|
| 41 | 4 000 | 80 | 14 | 15 | 7 | A | Continue normal |
| 42 | 4 000 | 80 | 14 | 15 | 5 | A | Continue normal |
| 43 | 4 000 | 80 | 14 | 15 | 7 | A | Continue normal |
| 44 | 4 000 | 80 | 14 | 15 | 6 | A | Continue normal |
| 45 | 4 000 | 80 | 14 | 15 | 9 | A | Continue normal |
| 46 | 4 000 | 80 | 14 | 15 | 7 | A | Continue normal |
| 47 | 4 000 | 80 | 14 | 15 | 9 | A | Continue normal |
| 48 | 4 000 | 80 | 14 | 15 | 8 | A | Continue normal |
| 49 | 4 000 | 80 | 14 | 15 | 6 | A | Continue normal |
| 50 | 4 000 | 80 | 14 | 15 | 5 | A | Continue normal |
| 51 | 4 000 | 80 | 14 | 15 | 8 | A | Continue normal |
| 52 | 4 000 | 80 | 14 | 15 | 4 | A | Continue normal |
| 53 | 4 000 | 80 | 14 | 15 | 3 | A | Continue normal |
| 54 | 4 000 | 80 | 14 | 15 | 1 | A | Continue normal |
| 55 | 4 000 | 80 | 14 | 15 | 3 | A | Switch to reduced |

AQL = 10 % nonconforming, inspection level I (see example 33)

A = acceptable

The reduced sampling plans have the unusual feature of a gap between the acceptance and rejection numbers. The rules are that if the observed number of nonconforming items is equal to the acceptance number or less, the lot is accepted and reduced inspection is continued (provided that other conditions do not call for normal inspection). If the rejection number is reached or exceeded, the lot is not accepted and normal inspection is restored as from the next lot. If, however, the result falls in the gap between the acceptance and rejection numbers, the lot is accepted but normal inspection shall be restored [**11.1.4**].

The sample sizes for reduced inspection will be seen to follow the same series of numbers as for normal inspection, but set two stages lower. This again gives constancy on diagonals.

It will be noticed that no OC curves are given for reduced inspection. This is deliberate. There are two reasons for not giving them. The first is that they tend to be misleading in that the eye interprets the entire curve, whereas the right-hand end of the curve is largely irrelevant, as reduced inspection is permitted only when the percentage of nonconforming items is known to be smaller than the AQL from past evidence and there is good reason to expect the good quality to continue.

The second reason is that the vertical scale of the curves represents "Percentage of lots expected to be accepted", and this is rather meaningless for reduced inspection, because as soon as any lot is not accepted normal inspection is restored.

Sometimes reference to Table VIII will disclose an asterisk instead of an entry. This means that the number of sample items from the last 10 lots is not a sufficient basis on which to judge whether reduced inspection is allowable, in which case a greater number than 10 lots may be considered until an entry is found in the table. It will be seen that the first entry found in these circumstances is always 0, so this procedure is worth adopting only if no nonconforming items have been observed over the samples from more than 10 successive lots.

EXAMPLE 34

Table 10 continues the example of Table 9, 20 lots having been accepted on reduced inspection from lot 56 to lot 75. The reduced plan in use is found from Table II-C to be:

sample size $n$ = 32 items

acceptance number Ac = 7 nonconforming items

rejection number Re = 10 nonconforming items

As far as lot 81, 7 nonconforming items or fewer are found in each sample and reduced inspection continues, but the 9 nonconforming items of lot 82 call for a restoration of normal inspection even though the lot is accepted. Only 3 lots later, tightened inspection has to be imposed as two lots (83 and 85) from the last five in normal inspection have not been accepted. As reduced inspection is optional, an alert inspector would have restored normal inspection at lot 79, having observed 14 nonconforming items in 96 items sampled from lots 76, 77 and 78 from which it appears that the 10 % AQL is no longer being achieved.

**Table 10 — Ten lots from a hypothetical inspection process**

| Lot number | Lot size | Sample size | Ac | Re | Nonconforming items | Acceptability | Future action |
|---|---|---|---|---|---|---|---|
| 76 | 4 000 | 32 | 7 | 10 | 5 | A | Continue reduced |
| 77 | 4 000 | 32 | 7 | 10 | 2 | A | Continue reduced |
| 78 | 4 000 | 32 | 7 | 10 | 7 | A | Continue reduced |
| 79 | 4 000 | 32 | 7 | 10 | 3 | A | Continue reduced |
| 80 | 4 000 | 32 | 7 | 10 | 1 | A | Continue reduced |
| 81 | 4 000 | 32 | 7 | 10 | 4 | A | Continue reduced |
| 82 | 4 000 | 32 | 7 | 10 | 9 | A | Restore normal |
| 83 | 4 000 | 80 | 14 | 15 | 17 | N | Continue normal |
| 84 | 4 000 | 80 | 14 | 15 | 12 | A | Continue normal |
| 85 | 4 000 | 80 | 14 | 15 | 15 | N | Switch to tightened |

AQL = 10 % nonconforming, inspection level I (see example 34)

A = acceptable N = not acceptable

It should be noted that reduced inspection is not the only procedure that may be used when the producer's quality level is better than the AQL. Sometimes skip-lot sampling procedures might be advantageous (see ISO 2859-3). When the producer's process quality control is satisfactory and his quality level is significantly better than the AQL, indirect inspection may be applicable. The purchaser can then replace his acceptance sampling by the producer's inspection results.

## 3.16 Double and multiple sampling

The principles for selecting double or multiple plans from the tables are similar to those for single sampling, but table III or IV of the master tables is used instead of Table II, or the appropriate part of the page if using the extended Tables X.

If the extended tables are used, care should be taken to see that the correct sample sizes are taken, as the tables give only the cumulative sizes. However, the plans all have the feature that successive samples are equal in size to the first sample and this rule is easily remembered.

Where the appropriate single sampling plan has an acceptance number of zero, or a sample size of 2, no double plan is available. Where the appropriate single sampling plan has acceptance number zero, or sample size 2, 3 or 5, no multiple plan is available. The alternative is either to use single sampling or the double or multiple plan for the next larger sample size that is available for the required AQL.

EXAMPLE 35

If the AQL is 0,40 and the code letter is G, Table III-A has an asterisk referring to a footnote. Either Table II-A may be used, in which case the plan would be

| | |
|---|---|
| sample size | $n$ = 32 items |
| acceptance number | Ac = 0 nonconforming items |
| rejection number | Re = 1 nonconforming item |

or the user may continue down the 0,40 column in Table III-A until he finds the double plan under code letter K:

| | **First** | **Second** | **Combined** |
|---|---|---|---|
| sample size | 80 | 80 | 160 |
| acceptance number | 0 | 1 | |
| rejection number | 2 | 2 | |

If the extended tables are used, the same alternatives will be found.

For double or multiple sampling with reduced inspection, a result falling in the gap between acceptance and rejection numbers on any sample but the last means that a further sample should be taken, just as for normal or tightened inspection, but now there is also a gap between the final acceptance number and rejection number. A result in this gap means that the lot should be accepted but normal inspection restored, as with reduced single sampling.

Table IX gives "average sample size" curves for double and multiple sampling, which may be used to decide whether the gain from the use of double or multiple sampling, instead of single sampling, will be sufficient to be worthwhile [**12.5**].

The curves are classified by the value of the single sampling acceptance number, and are necessarily approximate to some extent, as they cannot apply exactly to all the different plans given. The horizontal scale of each curve is in units of "*n* times proportion nonconforming," where *n* is the sample size of the relevant single sampling plan. In any particular case, this scale may be divided by *n* to obtain a scale of proportion nonconforming.

The vertical scale is in terms of the same value of *n*. The line at the top of each diagram therefore represents the single sample size and the efficiency of the double and multiple plans may be judged from their curves in relation to this top line.

It should be noted that, in operating sampling inspection, it is expected that normal inspection with the submitted quality better than the AQL will be in force most of the time. In this case, the most relevant parts of these curves are the sections to the left of the arrows on the baseline. Those diagrams that have no arrows refer to acceptance numbers used only in tightened inspection.

When the single sampling plan has an acceptance number of 1, the multiple plan is, much of the time, less efficient than the double plan. It was impossible to avoid this regrettable feature without losing other valuable features of the tables. In these circumstances, double sampling is to be preferred unless there is some good reason, other than the average sample size, for desiring to use multiple.

Table IX assumes that curtailment of inspection, as described in **2.22**, is not used.

EXAMPLE 36

The single sampling plan for code letter K and an AQL of 2,5 % nonconforming is in use, namely:

| | |
|---|---|
| sample size | *n* = 125 items |
| acceptance number | Ac = 7 nonconforming items |
| rejection number | Re = 8 nonconforming items |

Consideration is being given to a possible change to double or multiple sampling.

The appropriate diagram in Table IX is that labelled Ac = 7, which is the acceptance number. If so desired, the bottom scale may be divided by 125 (the sample size) and multiplied by 100 to obtain a scale of percent nonconforming. The numbers 3, 6, 9 and 12 then become 2,4 %, 4,8 %, 7,2 % and 9,6 % nonconforming. Usually, however, it is not necessary to do this to discover the information sought.

Similarly, the scale on the left-hand side can be read as 0,25, 0,5 and 0,75 of 125, if desired.

Looking at the curves for this example, it will be seen that

a) the double plan always has a smaller average size than the single one, and the multiple plan always a smaller average than the double;

b) if the quality is perfect, the double sample size is about 2/3 of the single, the multiple about 1/4 of the single;

c) at the AQL these fractions have risen to about 7/10 and 6/10 respectively;

d) the maximum average value of the double plan is a little over 9/10 of the single; the maximum average value of the multiple plan a little over 8/10 of the single.

## 3.17 Limiting quality and the isolated lot

In the ISO 2859-1 scheme, it is assumed that a series of lots is to be offered for acceptance and hence that it is the upper end of the OC curve that is the more important. However, where the product consists of a single isolated lot, or a very short run of lots, the lower end of the curve is also of importance as it indicates the chance of accepting a single bad lot if one is offered among a series of good lots. This is particularly relevant when a single lot is purchased from among a series of lots.

It is for such cases that Tables VI-A, VI-B, VII-A and VII-B are designed [**12.6**]. Tables VI-A and VII-A express the limiting quality in percent nonconforming and Tables VI-B and VII-B in nonconformities per 100 items. It is necessary to separate the two in this instance, as it is at the lower end of the curve that they give somewhat different answers.

The values tabulated are $LQ_{10}$ and $LQ_5$ (see **2.8**), the subscripts denoting the consumer's risk in percent nonconforming.

The values in the LQ tables can be read also from the tabulated OC curves in the extended tables, but it is convenient to have them gathered together.

The tables refer to single sampling, but the figures apply approximately also to the equivalent double and multiple plans.

EXAMPLE 37

An isolated lot is to be inspected. It has been decided that a high probability of acceptance will be required if the quality of the lot is as good as 1,0 % nonconforming, but that there should be only a 10 % probability of acceptance if its quality should be as bad as 4 % nonconforming. Subject to these conditions, the smallest sample size available in the tables is required.

Referring to Table VI-A and to the AQL = 1,0 column, seek from the top downwards until a number is found equal to or less than 4,0. Code letter M is the first that satisfies the conditions, with an LQ value of 3,7 % nonconforming, and reference to the extended tables gives the required plan and its OC curve, namely:

| | |
|---|---|
| sample size | *n* = 315 items |
| acceptance number | Ac = 7 nonconforming items |
| rejection number | Re = 8 nonconforming items |

It is as well at this point to re-iterate the meaning of the OC curve. The LQ value of 3,7 % nonconforming means that if the lot has 3,7 % nonconforming, there will be a 10 % chance of accepting. It does not mean that there is a 10 % chance that the lot will be 3,7 % nonconforming.

It will be noticed that LQ values are always greater than the AQL, and in some cases considerably greater, but the difference between the LQ and the AQL values decreases as the sample size increases.

Where an isolated lot is concerned, as distinct from a continuing series of lots, the LQ values should be regarded as only approximate if the sample size is more than 1/5 of the lot size. Under these circumstances, the true value will be rather less than the tabulated value.

ISO 2859-2 contains further details of the method of sampling for lots in isolation.

## 3.18 Sample sizes

The sample sizes given in ISO 2859-1 for single sampling form a series (like the series of AQL values) in which each number is about 1,585 times the preceding one. This means that the product "AQL times sample size" is approximately constant on the diagonals from bottom left to top right of Table II-A, which leads to a self-consistent table if acceptance numbers are also taken as constant on the diagonals.

This feature was helpful in designing the tables rather than being directly helpful in using them, but the resulting pattern does mean that the tables lend themselves to the construction of convenient summaries and of special nomograms or slide-rules that could be convenient on occasions. See, for example, **3.21** and Figure 12 and Figure 13.

The sample sizes for double and multiple sampling (see **2.20** and **2.21**) follow the same pattern, but for a given code letter the double sample size is stepped back one place in the series compared with the single, whereas the multiple sample size is stepped back two more places beyond the double. Sample sizes for reduced inspection are always stepped back two places compared with the corresponding normal inspection.

As a result, five different values of sample size correspond to any given code letter according to whether single, double or multiple sampling is used, and to whether or not reduced inspection (see **3.15**) is in force. This is why code letters are needed to index the tables, rather than purely sample sizes.

## 3.19 Operating characteristic curves

The extended Tables X of ISO 2859-1 give both the drawings of OC curves and the tabulated values from which the drawings were made.

They apply to single sampling, but the curves for double and multiple sampling have been approximately matched [**12.1**].

A study of the OC curves given in ISO 2859-1 will show that when the acceptance number is zero the upper end of the curve is rather difficult to read accurately. There is, however, a simple approximate formula for this upper end when the acceptance number is zero, which is accurate enough for practical purposes whatever the sample size.

The formula is

Percentage of lots expected to be accepted
= 100 – $n$(percent nonconforming in submitted lots)

Note that this formula is valid only for acceptance number zero, and only for the upper end of the curve, i.e. where more than 80 % of the lots are expected to be accepted.

EXAMPLE 38

Suppose AQL is 0,40 % nonconforming with code letter G.

The sampling plan is:

| | |
|---|---|
| sample size | $n$ = 32 items |
| acceptance number | Ac = 0 nonconforming items |
| rejection number | Re = 1 nonconforming item |

What is the percentage of lots expected to be accepted at the AQL?

The answer is

100 – (32 × 0,40) % of lots = 87,2 % of lots

This value is close to the exact value, 88,0 %, which is calculated from $100(1 - 0{,}0040)^{32}$.

EXAMPLE 39

In the same circumstances what would have to be the percent nonconforming in submitted lots for 95 % of lots to be accepted?

The previous formula written in a different way gives

Percent nonconforming in submitted lots =

$$\frac{(100 - \text{percentage of lots expected to be accepted})}{\text{Sample size}}$$

$$= \left(\frac{100 - 95}{32}\right) \% \text{ nonconforming}$$

= 0,156 % nonconforming

This value is close to the 0,160 % given in Table X-G-1 of ISO 2859-1.

## 3.20 The AOQL table (see 2.12 for theory)

Tables V-A and V-B give AOQL factors for the normal and tightened single sampling plans. They also apply approximately to the equivalent double and multiple plans.

If more accurate values of AOQL are desired, then multiply the tabulated value by the following correction factor, $f$.

a) In the case of inspection by percent nonconforming

$$f = 1 - \frac{2n}{3N} \text{ (for Ac > 0)}$$

and

$$f = 1 - \frac{n}{2N} - \frac{1}{2n} \text{ (for Ac = 0)}$$

Where

$n$ is the sample size;

$N$ is the lot size.

b) In the case of inspection by nonconformities per 100 items

$$f = 1 - \frac{\text{Ac} \times n}{(2\text{Ac} + 3)N} \text{ (for all Ac values)}$$

NOTE 6 If the above correction factor, whichever is applicable, is close to 1,00, then it makes little difference and the tabular values may be used without any correction; otherwise this multiplication is desirable.

NOTE 7 In the case of inspection by nonconformities per 100 items, if Ac = 0 then no correction is necessary.

A study of Table V-B will show that, except in the top diagonal (where the acceptance number is 0), the AOQL for tightened inspection is always close to the AQL. If it is desired to keep this relationship between AQL and AOQL for tightened inspection, then use should be made of the option of using the plans with an acceptance number of 1 instead of those with an acceptance number of 0.

EXAMPLE 40

For a lot size of 400, an AQL of 4,0 % nonconforming and inspection level II, the code letter is found to be H. For normal inspection, the AOQL is found from Table V-A to be 6,3 %, and hence a more accurate value is

$$6{,}3 \times \left(1 - \frac{2 \times 50}{3 \times 400}\right) = 5{,}8 \text{ \% nonconforming}$$

NOTE 8 True values of AOQL can be obtained by means of a complicated calculation using the hypergeometric distribution. In the case of the above example, the true value of the AOQL is 5,809 %.

NOTE 9 In the case of the above example, if the inspection were by nonconformities per 100 items in place of by percent nonconforming, then the AOQL value would be as follows:

$$6{,}3 \times \left(1 - \frac{5 \times 50}{(10 + 3) \times 400}\right) = 6{,}0 \text{ (for Ac = 5)}$$

## 3.21 Nomograms

In designing the tables of ISO 2859-1, certain mathematical relationships were used that enable some features of the tables to be set out in a simplified form as shown in Figure 12 and Figure 13.

These diagrams do not replace the tables, but they may be interesting in showing the relationship between the various figures, and sometimes useful in giving some of the information from the tables in a much condensed form.

To use Figure 12, suppose one wishes to know what sample size (for single sampling and normal inspection) corresponds to code letter H. A straight line across the figure from the point labelled H on the left-hand scale to the point labelled Single (Normal or Tightened) on the right-hand scale, cuts the centre scale at the point labelled 50, which is therefore the required sample size.

NOTE 10 Rather than actually drawing lines on the figure, it is better to use the edge of a ruler, or a stretched length of cotton, to keep the page clean for future use.

Similarly in Figure 13, if one wishes to know the acceptance number that goes with a sample size of 50 for an AQL of 2,5, a straight line cuts the centre scale at the point labelled 3 for normal inspection, or 2 for tightened inspection.

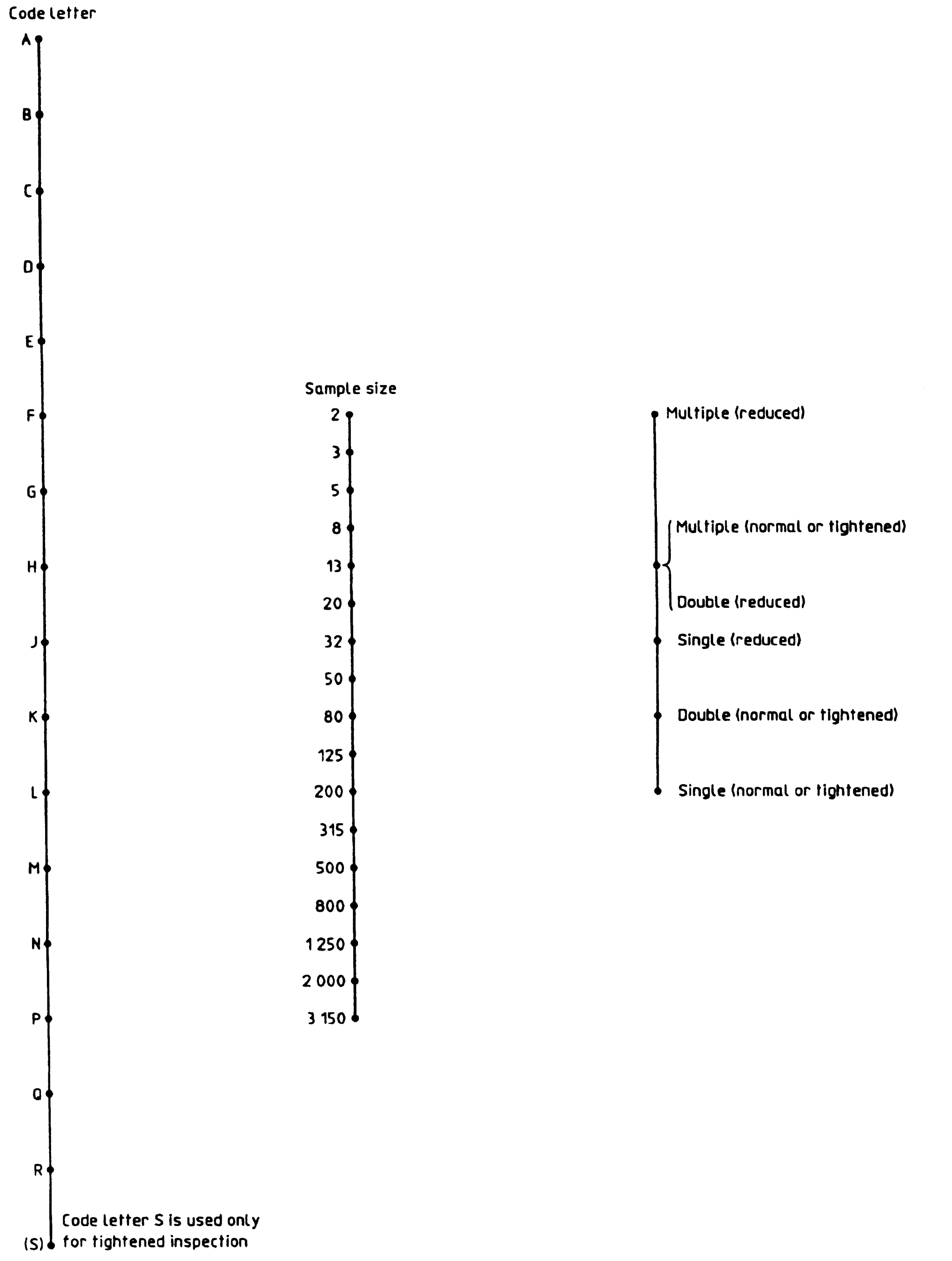
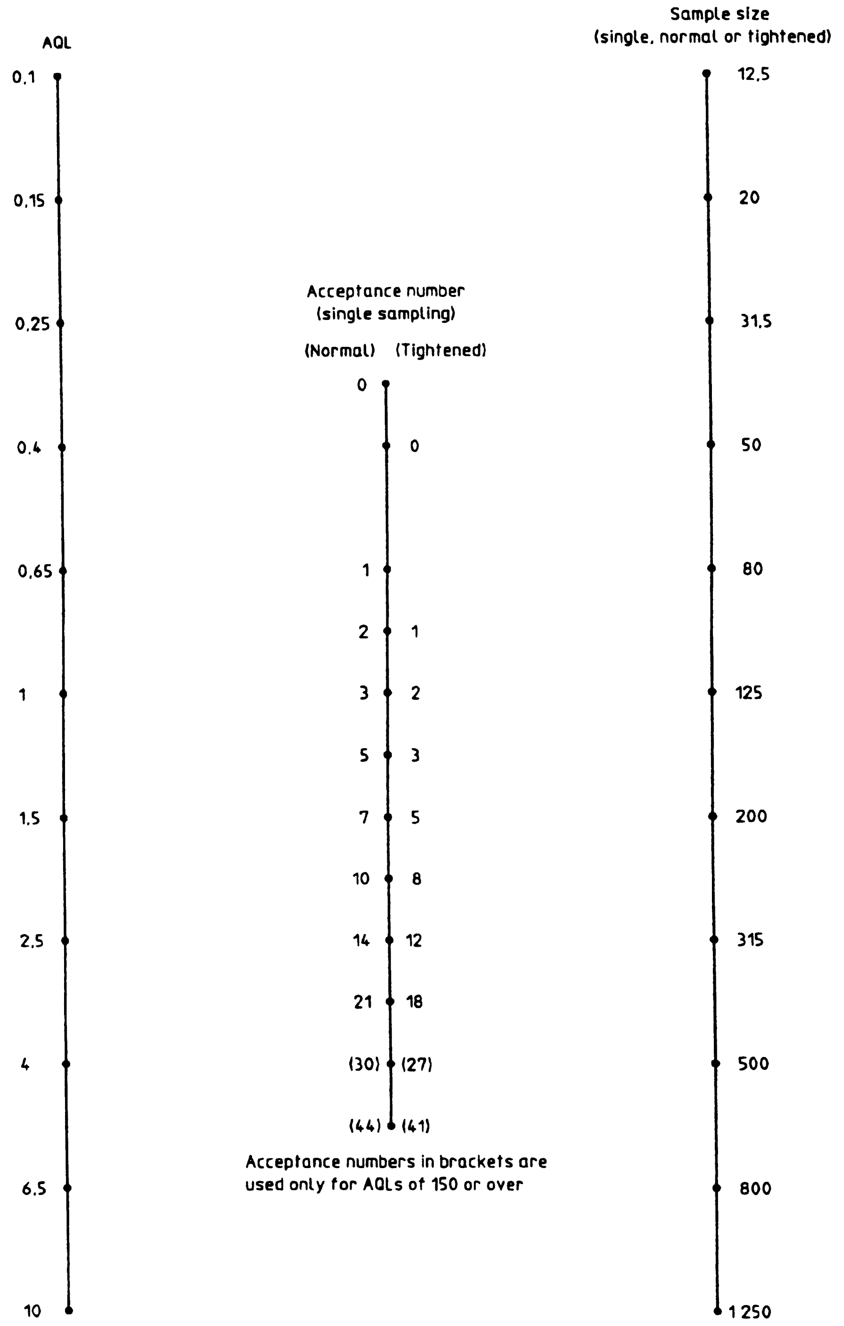



**Figure 12 — Nomogram for type of sampling, code letter and sample size in ISO 2859-1**




NOTE — AQL may be multiplied by 10 if the sample size is divided by 10, and vice versa, similarly for any power of 10. Sample size is to be rounded to nearest whole number.

**Figure 13 — Nomogram for AQL, sample size and acceptance number in ISO 2859-1**

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# List of references

See national foreword.

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